

Fizika I

KINEMATIKA ČESTICE 2. dio

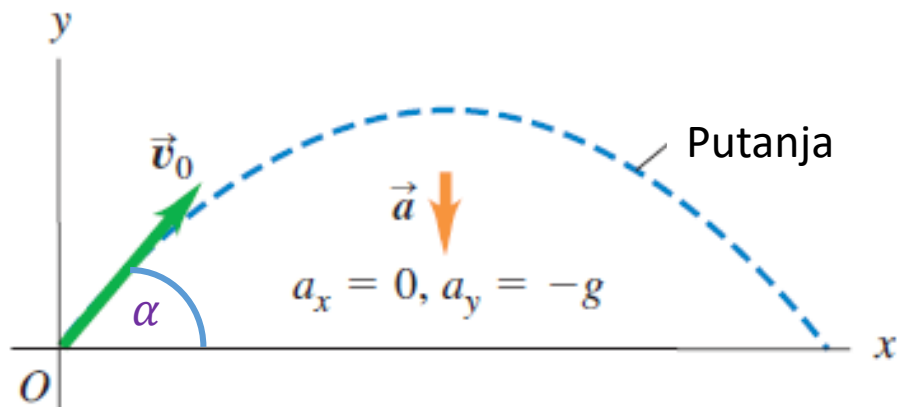
Doc. dr. sc. Iva Movre Šapić

Krivocrtno gibanje

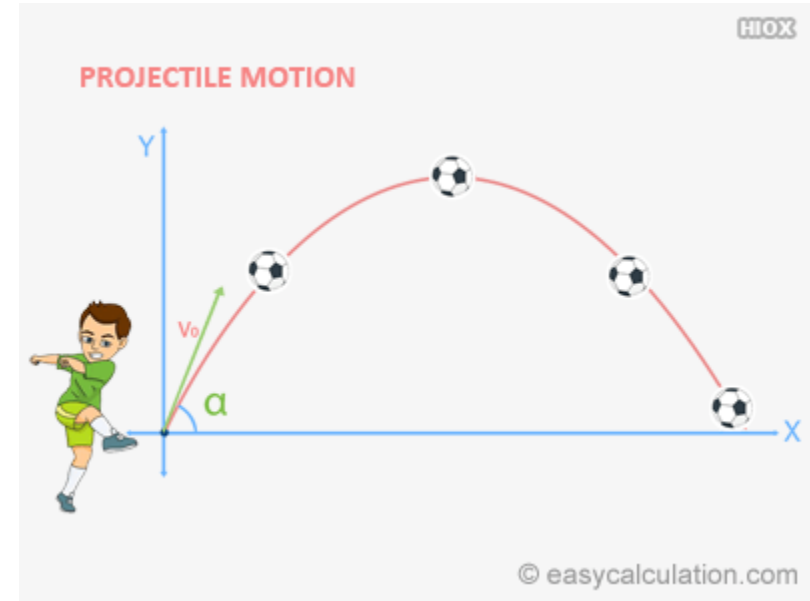
Kad ubrzanje materijalne točke nema isti smjer kao i brzina, materijalna točka giba se po **zakrivljenoj putanji**.

Kosi hitac

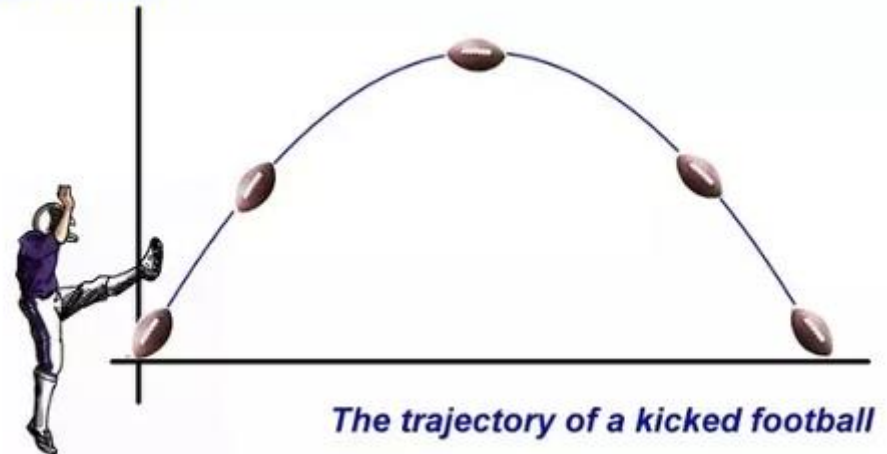
- Tijelo je izbačeno početnom brzinom $\vec{v}_0$ (v_0, α) i dalje se giba samo pod djelovanjem akceleracije gravitacije g (i otpora zraka što se često može zanemariti)
- Gibanje se odvija u vertikalnoj ravnini koju definira vektor početne brzine $\vec{v}_0$



Kosi hitac - primjeri

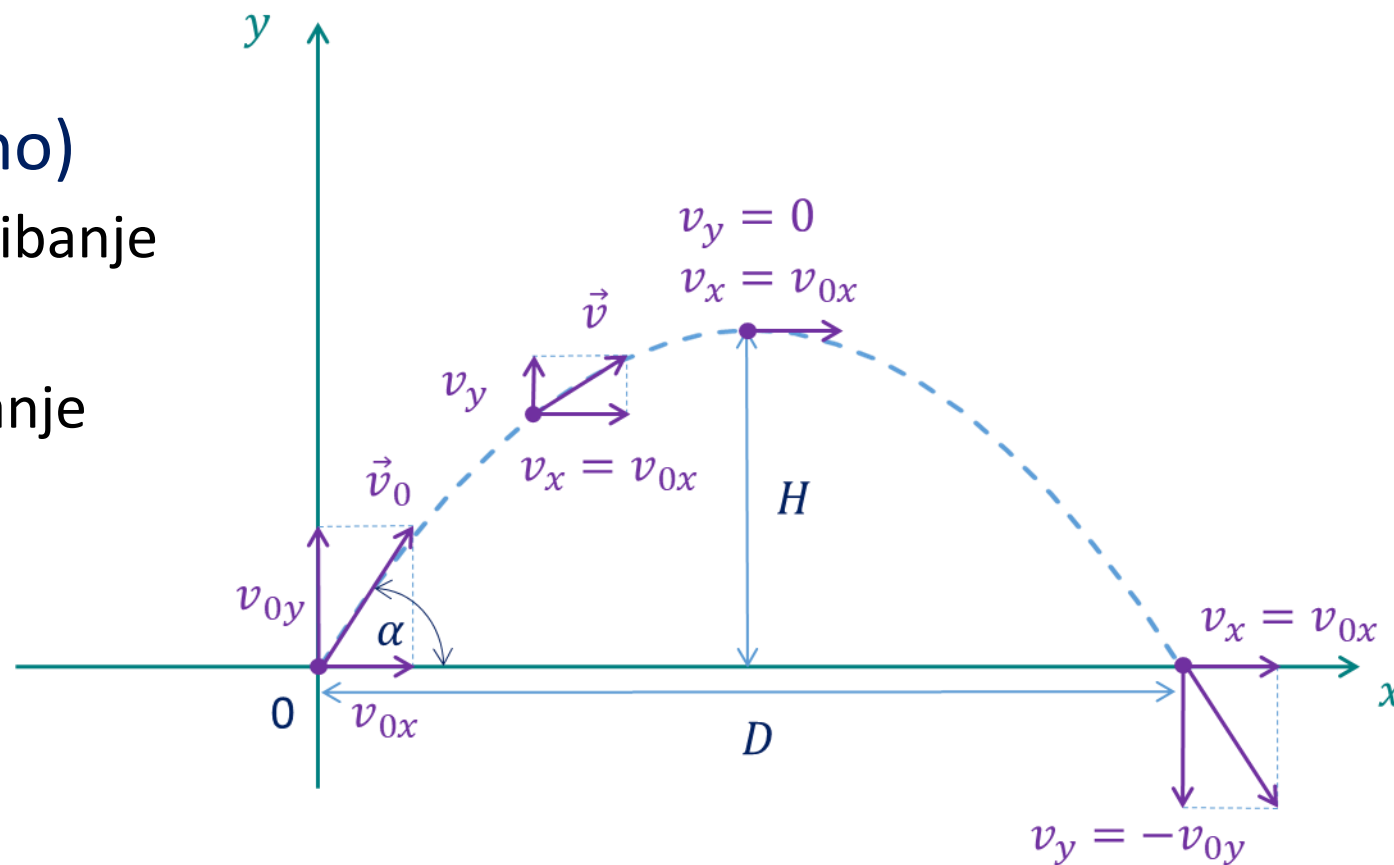


Projectile Motion

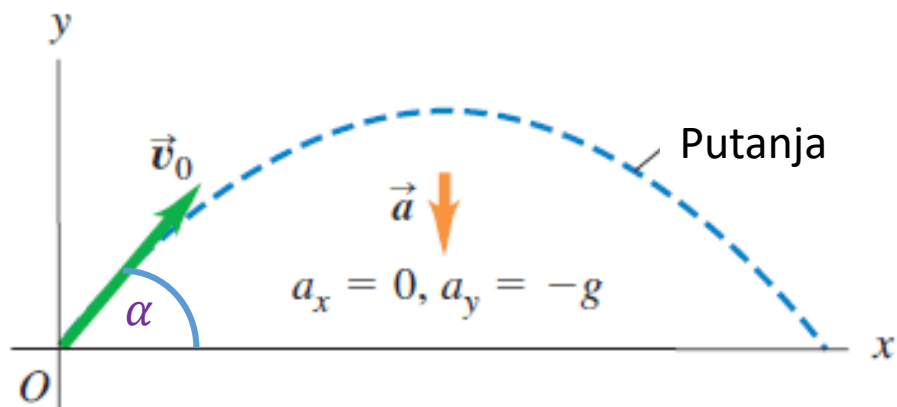


Kosi hitac

- Složeno gibanje: (istovremeno)
 - x – os: jednoliko pravocrtno gibanje
($v_x = v_{0x}$)
 - y – os: jednoliko ubrzano gibanje
($a_y = -g$)
 - t – zajednička varijabla



Kosi hitac



$$a_x = 0$$

$$a_y = -g$$

Početni trenutak

$$v_x(t = 0) = v_{0x} = v_0 \cos \alpha$$

$$v_y(t = 0) = v_{0y} = v_0 \sin \alpha$$

U svakom trenutku, t

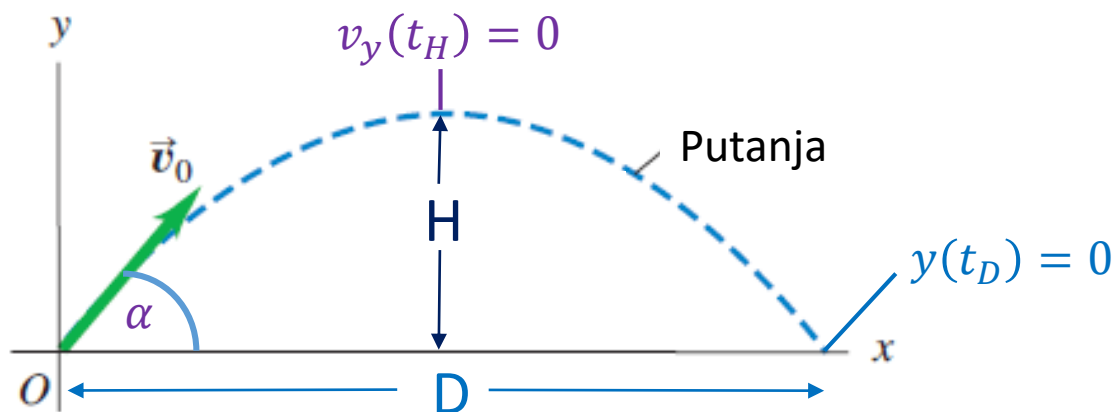
$$v_x(t) = v_{0x}$$

$$v_y(t) = v_{0y} - gt$$

$$x(t) = v_{0x}t$$

$$y(t) = y_0 + v_{0y}t - \frac{1}{2}gt^2$$

Kosi hitac



$$\begin{aligned}x(t) &= v_{0x}t \\v_x(t) &= v_{0x}\end{aligned}\qquad\qquad\begin{aligned}y(t) &= y_0 + v_{0y}t - \frac{1}{2}gt^2 \\v_y(t) &= v_{0y} - gt\end{aligned}$$

- putanja:

$$y = x \tan \alpha - \frac{g}{2v_0^2 \cos^2 \alpha} x^2$$

- maksimalna visina: $v_y(t_H) = 0$

$$H = y(t_H) = \frac{v_0^2 \sin^2 \alpha}{2g}$$

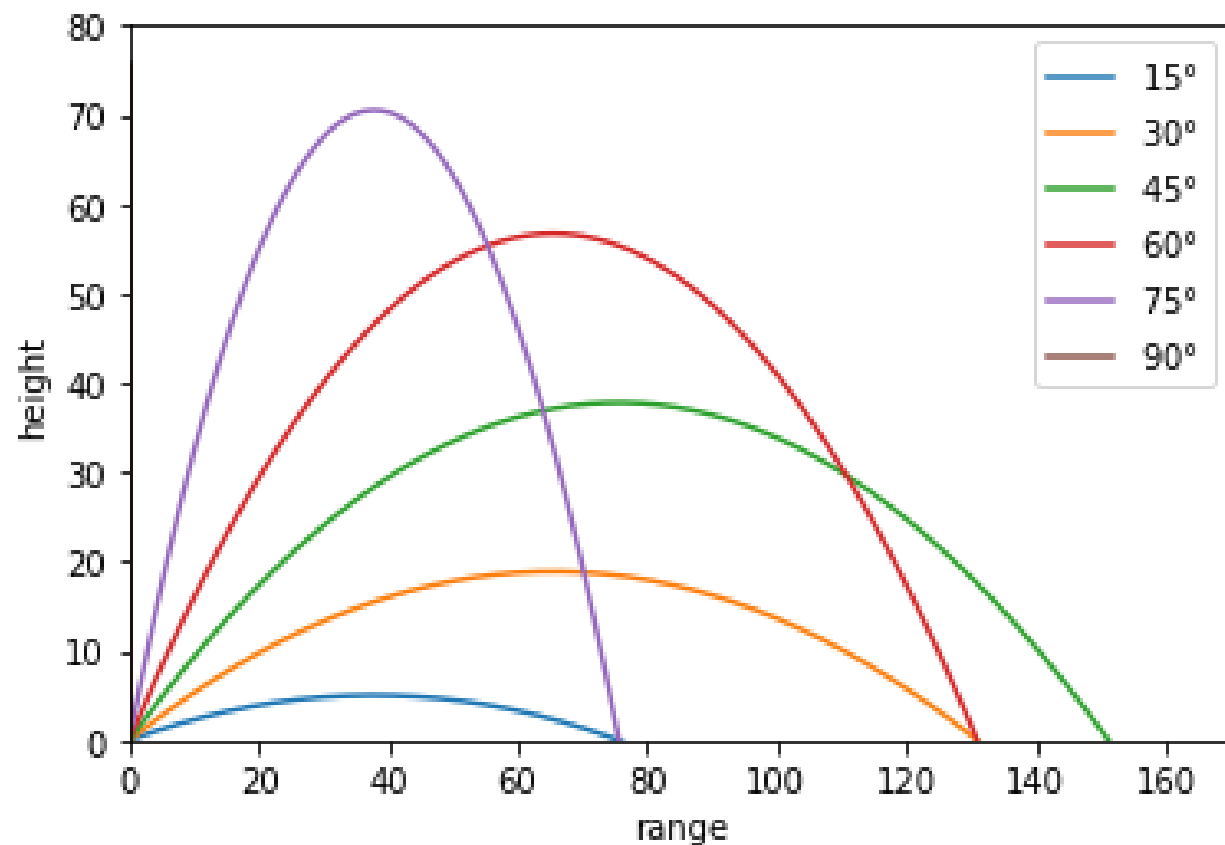
- domet: $y(t_D) = 0$

$$D = \frac{v_0^2 \sin 2\alpha}{g}$$

- vrijeme uspinjanja:

$$t_H = \frac{v_0 \sin \alpha}{g}$$

Kosi hitac – najveći domet



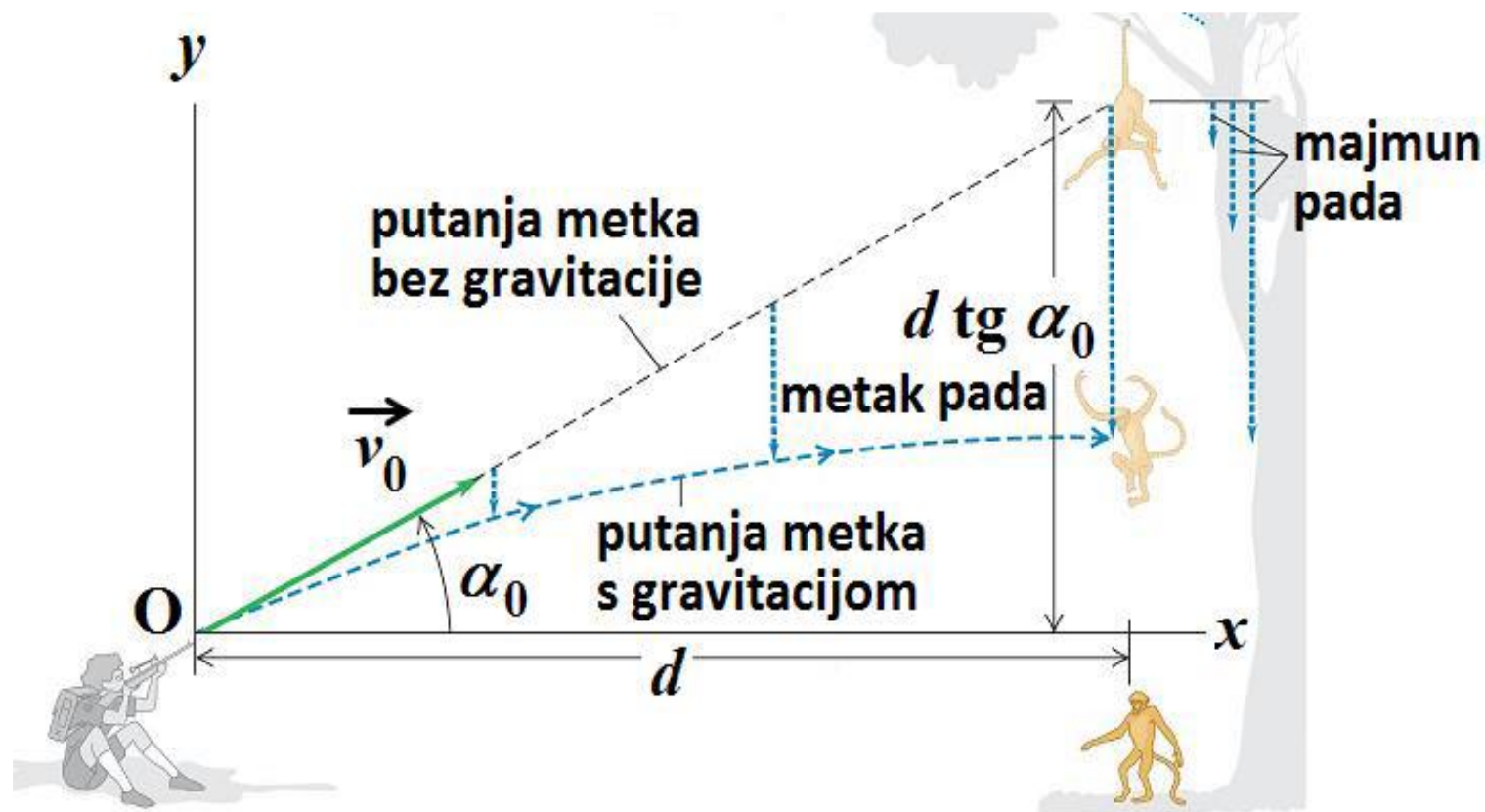
$$\frac{dD(\alpha_{max})}{d\alpha} = 0$$

$$\frac{2v_0^2 \cos 2\alpha_{max}}{g} = 0$$

$$\alpha_{max} = 45^\circ$$

$$D(\alpha) = D(90^\circ - \alpha)$$

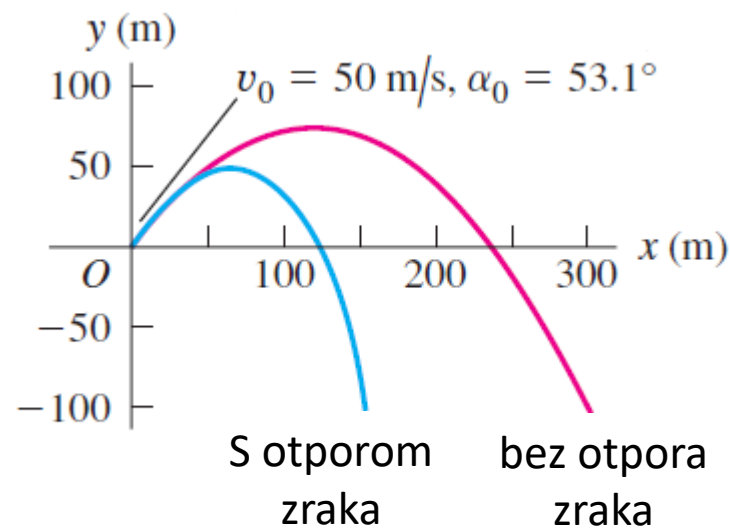
Kosi hitac



MIT Physics Demo - Monkey and a Gun

<https://www.youtube.com/watch?v=cxvsHNRXLjw>

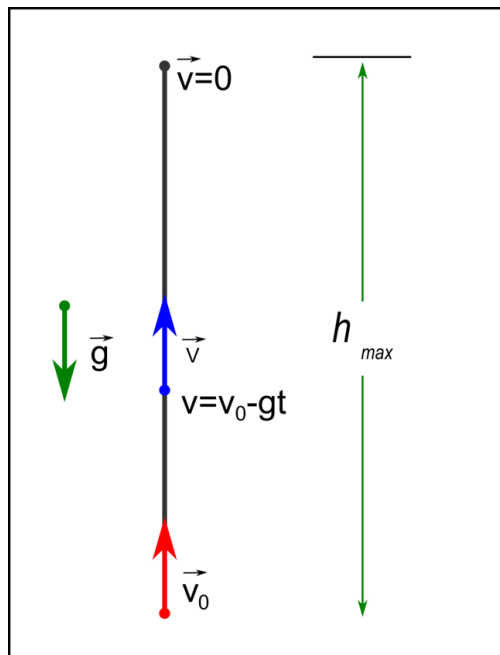
Balističke krivulje



- Otpor zraka nije zanemariv
- Putanja ovisi o obliku, presjeku i brzini projektila,
- Temperaturi, tlaku, vjetru...

Kosi hitac

- Posebni slučajeви:
 - $\alpha = 90^\circ$ vertikalni hitac prema gore
 - $\alpha = 270^\circ$ vertikalni hitac prema dolje
 - $\alpha = 0^\circ$ horizontalni hitac



Jednoliko
usporeno pa
jednoliko ubrzano
gibanje
(slobodni pad)

hitac u vis:

$$\alpha = 90^\circ$$

$$a = -g$$

$$v = v_0 - gt$$

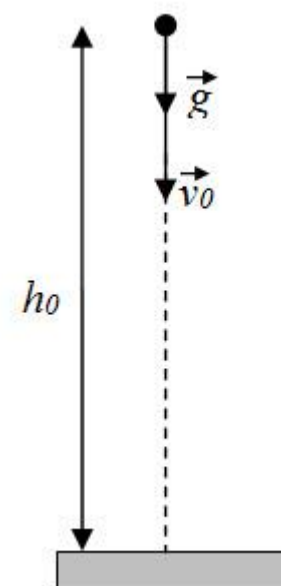
$$h = h_0 + v_0 t - \frac{1}{2} g t^2$$

$v = 0$ tijelo se zaustavlja! $h = h_{max}$

$$v_0 - gt = 0 \rightarrow t = \frac{v_0}{g}$$

Vertikalni hitac

Jednoliko ubrzano
gibanje



hitac prema dolje:

$$\alpha = 270^\circ$$

$$a = g$$

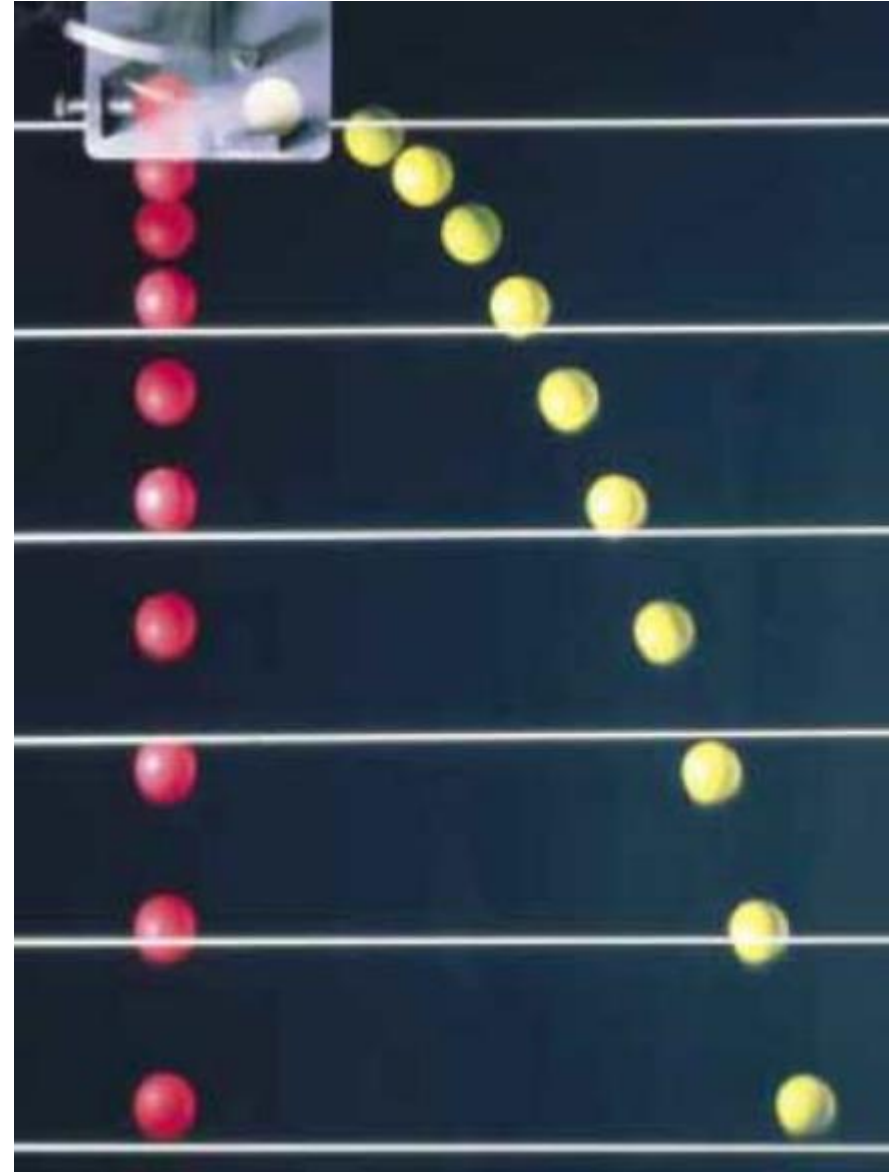
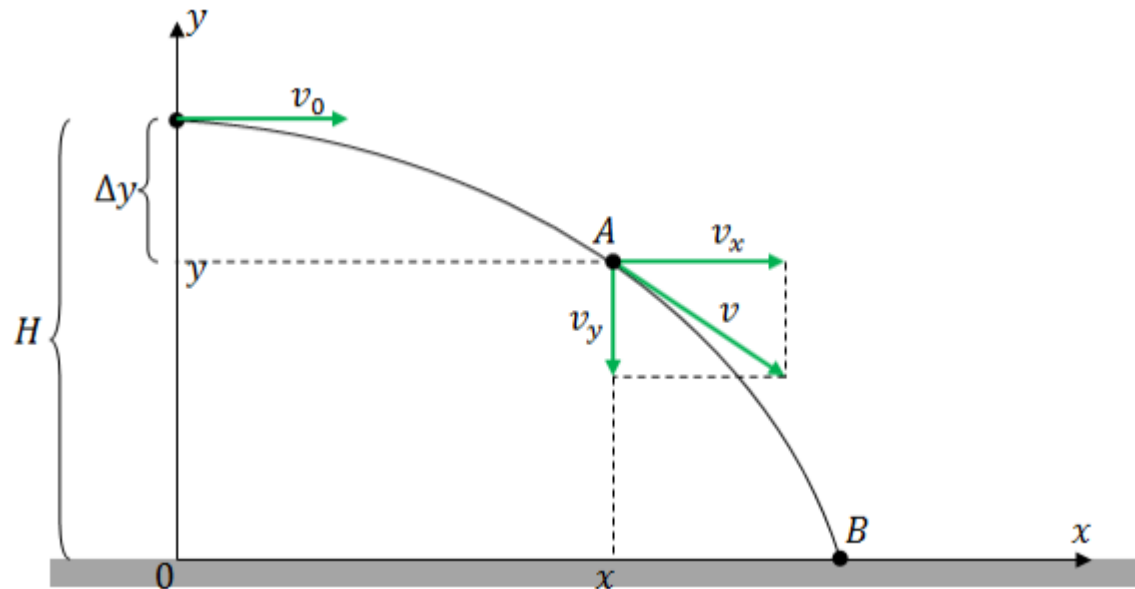
$$v = v_0 + gt$$

$$s = v_0 t + \frac{1}{2} g t^2$$

Ako je ishodište na tlu: $h = h_0 - v_0 t - \frac{1}{2} g t^2$

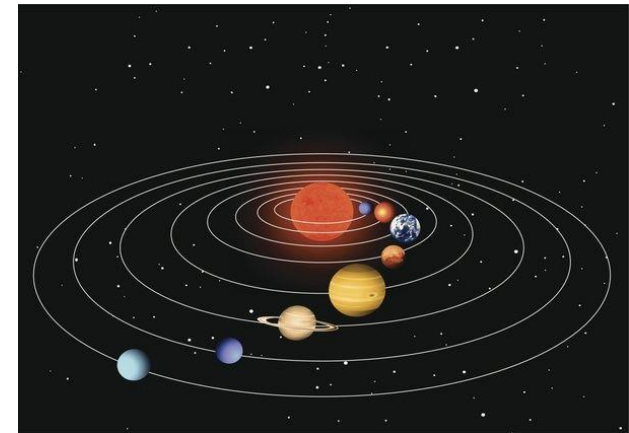
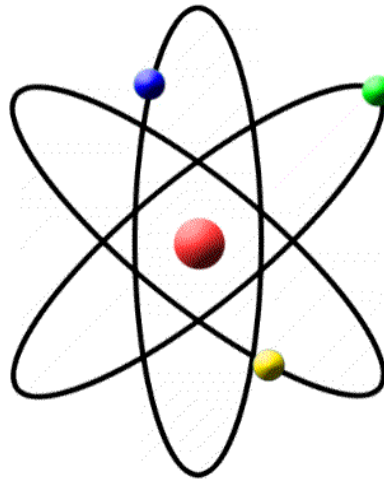
Horizontalni hitac

$$\alpha = 0^\circ$$

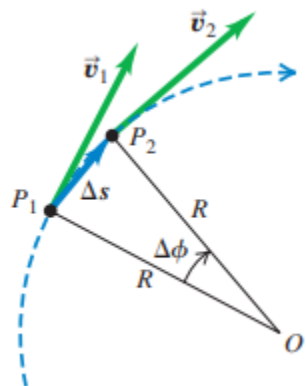


Jednoliko gibanje po kružnici

- najjednostavnije krivocrtno gibanje
 - Brzina je konstantna po iznosu, ali stalno mijenja smjer!



Jednoliko gibanje po kružnici



- Za vrijeme Δt materijalna točka prijeđe put Δs (dio kružnog luka), odnosno kut $\Delta\varphi$

$$\Delta s = r\Delta\varphi$$

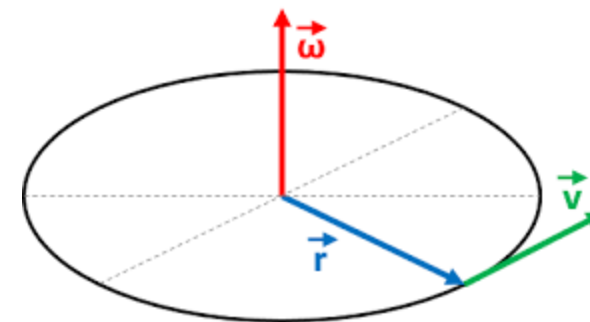
Linearna (obodna) brzina

$$v = \lim_{\Delta t \rightarrow 0} \frac{\Delta s}{\Delta t} = r \lim_{\Delta t \rightarrow 0} \frac{\Delta\varphi}{\Delta t} = r \frac{d\varphi}{dt} = r\omega$$

Kutna brzina

$$\omega = \frac{d\varphi}{dt} \quad [\text{rad/s}]$$

$$\vec{v} = \vec{\omega} \times \vec{r}$$



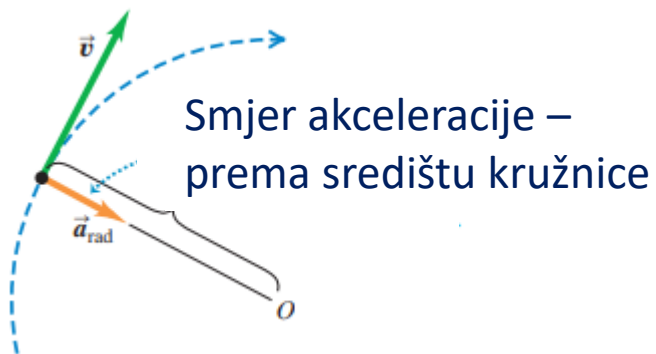
Jednoliko gibanje po kružnici

- Obodna brzina, iako ne mijenja iznos, mijenja smjer i zato je jednoliko kružno gibanje zapravo ubrzano gibanje.

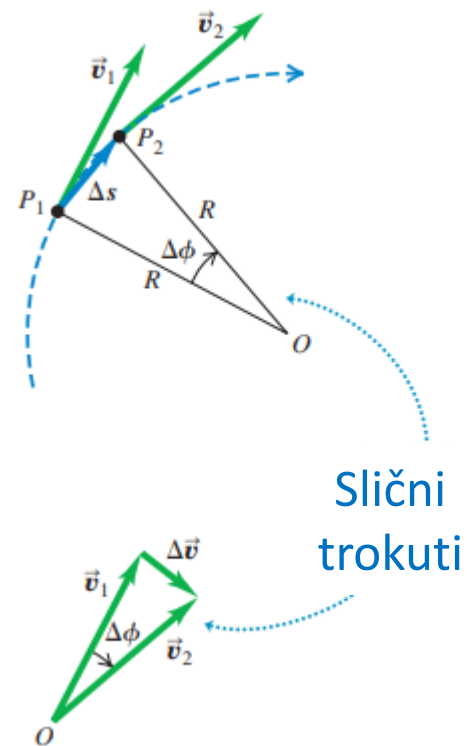
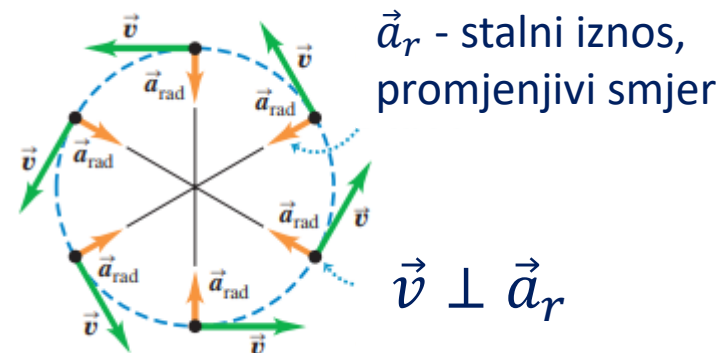
Radijalna (centripetalna) akceleracija

$$|\Delta \vec{v}| = v \Delta \phi \quad /: \Delta t$$

$$a_r = \lim_{\Delta t \rightarrow 0} \frac{|\Delta \vec{v}|}{\Delta t} = \lim_{\Delta t \rightarrow 0} \frac{v \Delta \phi}{\Delta t} = v \lim_{\Delta t \rightarrow 0} \frac{\Delta \phi}{\Delta t} = v \omega$$



$$\vec{a}_r = -r \omega^2 \vec{r}_0 = -\frac{v^2}{r} \vec{r}_0 = \vec{\omega} \times \vec{v}$$



Jednoliko gibanje po kružnici

- Kruženje konstantnom kutnom brzinom:

$$\frac{d\varphi}{dt} = \omega = \textit{konst.}$$

- Integriranjem dobijemo

$$\varphi = \omega t + \varphi_0$$

Uvodimo:

Ophodno vrijeme, T = vrijeme potrebno da materijalna točka jednom obiđe kružnicu

Frekvencija, f = broj okreta u sekundi

$$\omega = 2\pi f; \quad T = \frac{1}{f}$$

Nejednoliko kružno gibanje

Mijenja se i iznos obodne brzine s vremenom

→ imamo i **tangencijalnu akceleraciju**:

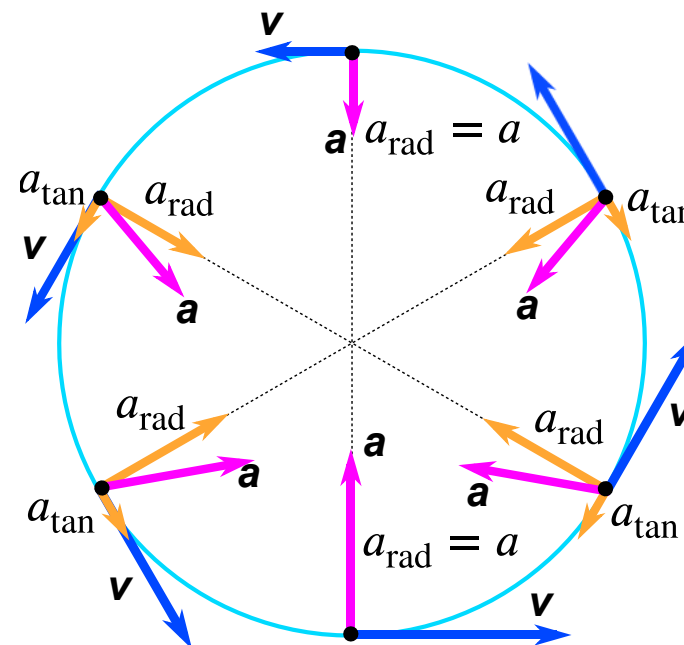
$$a_t = \frac{dv}{dt} = \frac{d(r\omega)}{dt} = r \frac{d\omega}{dt} = r\alpha$$

Kutna akceleracija:

$$\alpha = \lim_{\Delta t \rightarrow 0} \frac{\Delta\omega}{\Delta t} = \frac{d\omega}{dt} = \frac{d^2\varphi}{dt^2}$$

Ukupna akceleracija:

$$\vec{a} = \vec{a}_t + \vec{a}_r; \quad a = \sqrt{(a_t^2 + a_r^2)}$$



Posebni slučaj $\alpha = konst.$

Integriranje $d\omega = \alpha dt \rightarrow \omega = \alpha t + \omega_0$

$$d\varphi = (\alpha t + \omega_0) dt$$

Integriranjem $\rightarrow \varphi = \frac{\alpha}{2} t^2 + \omega_0 t$

Usporedba pravocrtnog i kružnog gibanja

Pravocrtno gibanje	Kružno gibanje
$a = \frac{dv}{dt} = \frac{d^2s}{dt^2}$	$\alpha = \frac{d\omega}{dt} = \frac{d^2\varphi}{dt^2}$
$v = \frac{ds}{dt}$	$\omega = \frac{d\varphi}{dt}$
$v = v_0 + at$	$\omega = \omega_0 + \alpha t$
$v^2 = 2as + v_0^2$	$\omega^2 = 2\alpha\varphi + \omega_0^2$
$s = s_0 + vt$	$\varphi = \varphi_0 + \omega t$
$s = s_0 + v_0t + \frac{1}{2}at^2$	$\varphi = \varphi_0 + \omega_0t + \frac{1}{2}\alpha t^2$