Fizika I

KINEMATIKA ČESTICE 2. dio

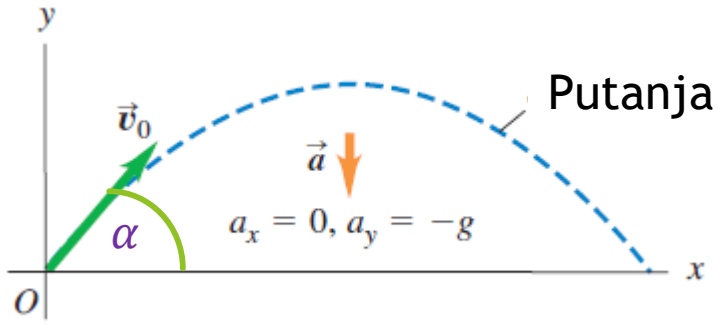

Gatorade

Krivocrtno gibanje

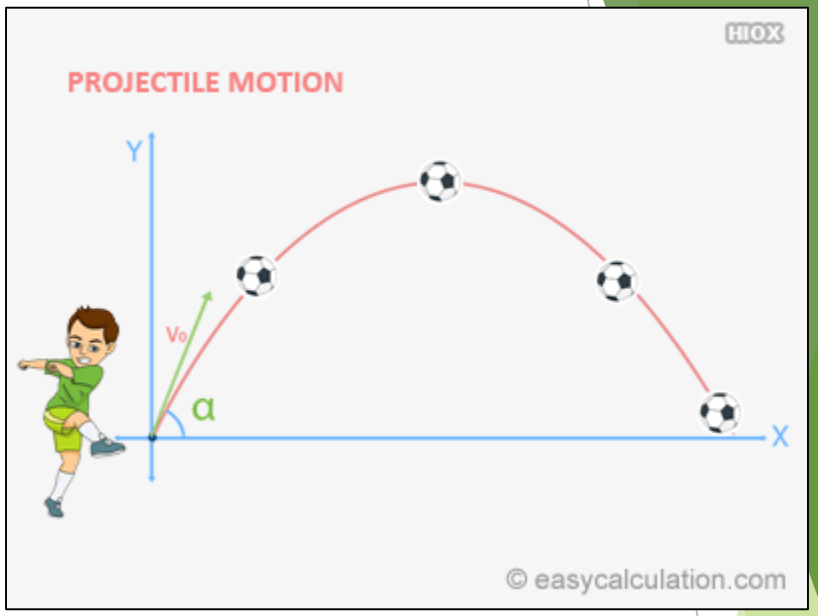
Kad ubrzanje materijalne točke nema isti smjer kao i brzina, materijalna točka giba se po zakrivljenoj putanji.

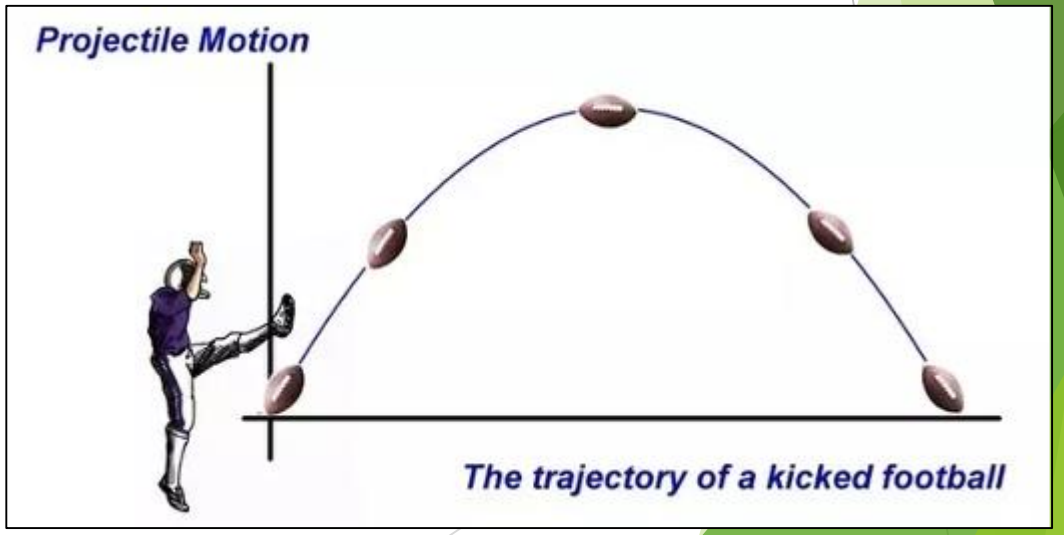
Kosi hitac

- Tijelo je izbačeno početnom brzinom $\vec{v}_0$ (v_0, α) i dalje se giba samo pod djelovanjem akceleracije gravitacije g (i otpora zraka što se često može zanemariti).
- Gibanje se odvija u vertikalnoj ravnini koju definira vektor početne brzine $\vec{v}_0$.



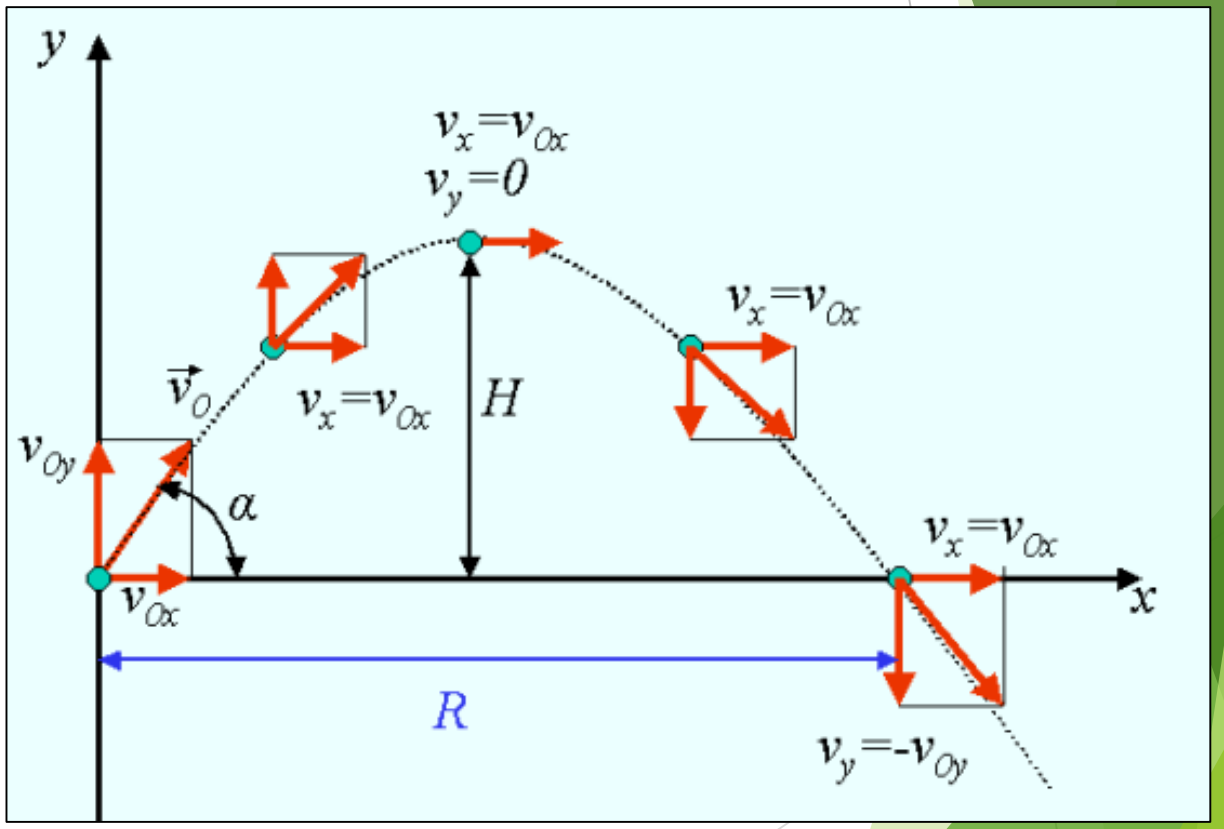
Kosi hitac - primjeri



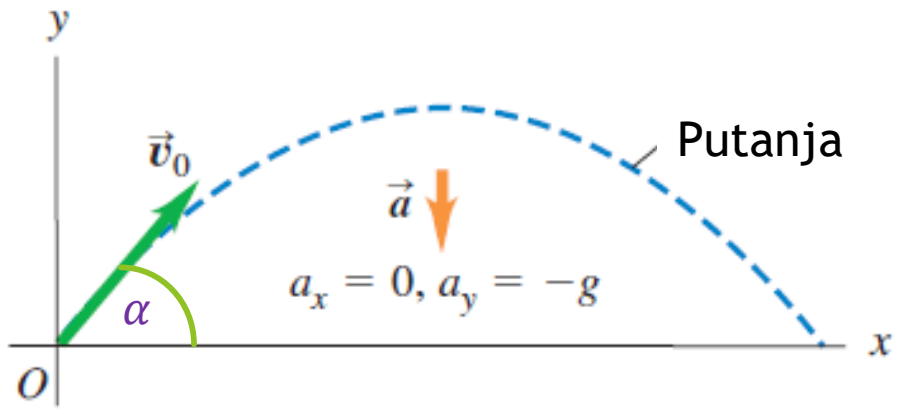


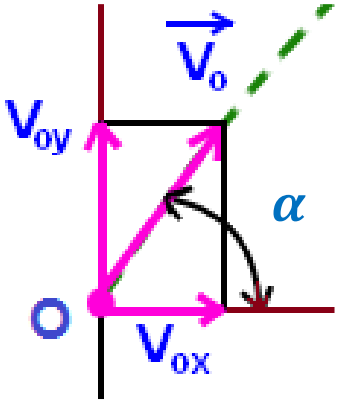
Kosi hitac

- Složeno gibanje: (istovremeno)
  - x – os: jednoliko pravocrtno gibanje
    $(v_x = v_{0x})$
  - y – os: jednoliko ubrzano gibanje
    $(a_y = -g)$
  - t – zajednička varijabla



Kosi hitac





$$a_x = 0 \qquad a_y = -g$$

Početni trenutak

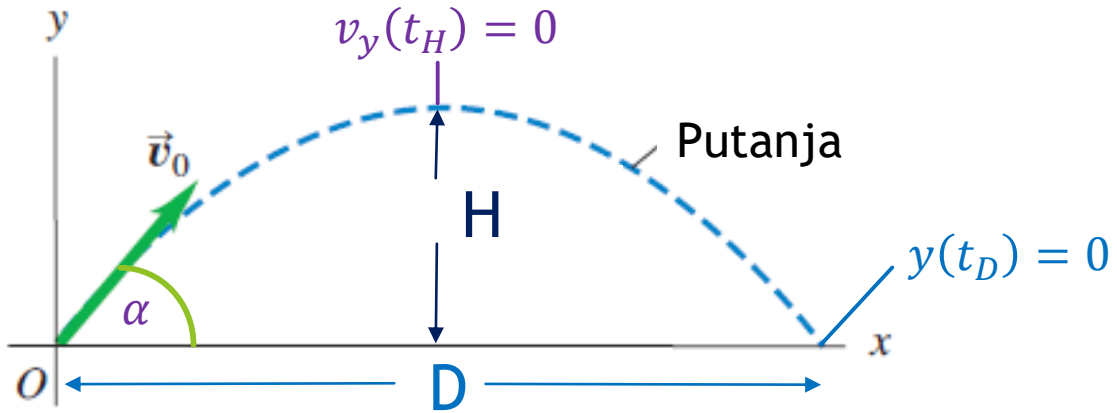
$$v_x(t=0) = v_{0x} = v_0 \cos\alpha \qquad v_y(t=0) = v_{0y} = v_0 \sin\alpha$$

U svakom trenutku, t

$$v_x(t) = v_{0x} \qquad v_y(t) = v_{0y} - gt$$

$$x(t) = v_{0x}t \qquad y(t) = y_0 + v_{0y}t - \frac{1}{2}gt^2$$

Kosi hitac

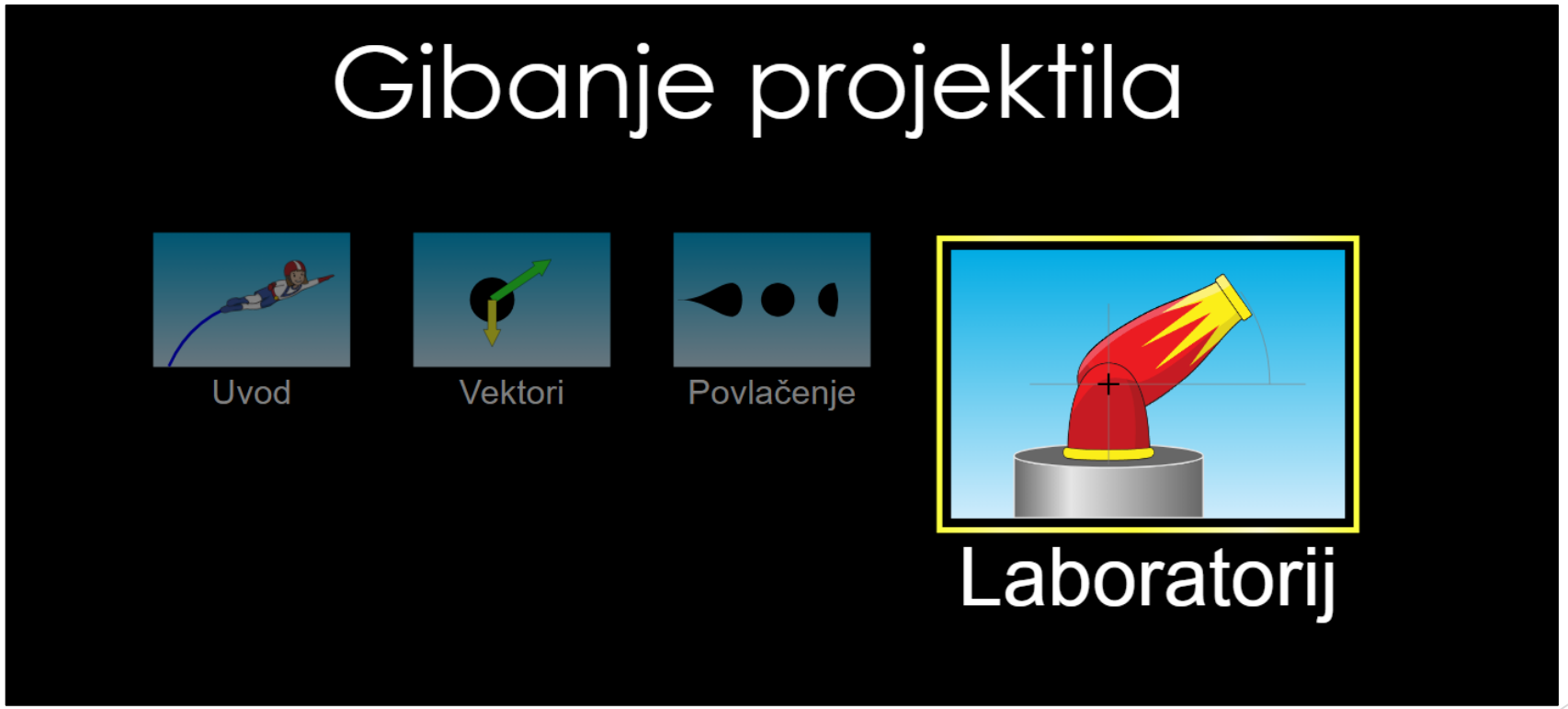


$$x(t) = v_{0x}t$$
$$v_x(t) = v_{0x}$$

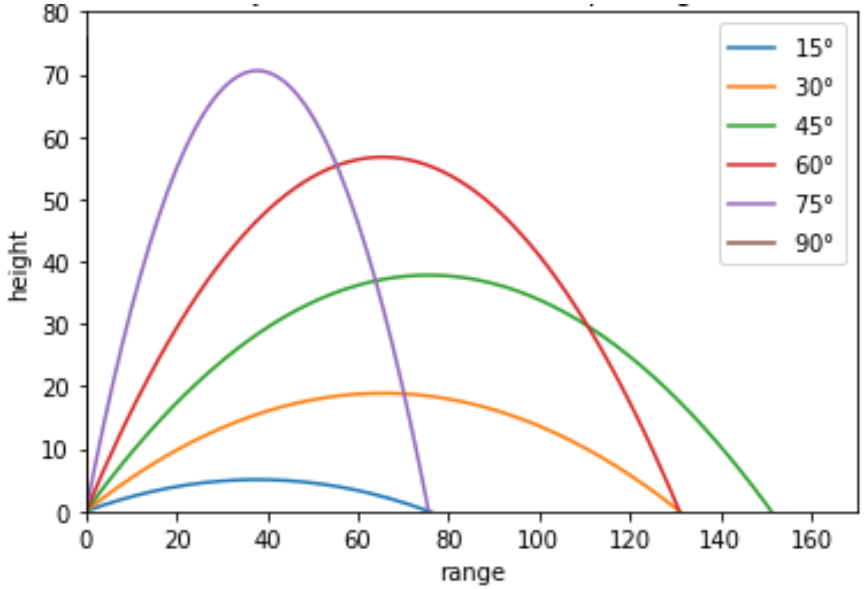
$$y(t) = y_0 + v_{0y}t - \frac{1}{2}gt^2$$
$$v_y(t) = v_{0y} - gt$$

- putanja: $y = x\tan\alpha - \frac{g}{2v_0^2\cos^2\alpha}x^2$
- maksimalna visina: $v_y(t_H) = 0$ $\quad \mathrm{H} = y(t_H) = \frac{v_0^2\sin^2\alpha}{2g}$
- domet: $y(t_D) = 0$ $\quad D = \frac{v_0^2\sin 2\alpha}{g}$
- vrijeme uspinjanja: $\quad t_H = \frac{v_0\sin\alpha}{g}$

Simulacija: Kosi hitac



Kosi hitac – najveći domet



$$D = \frac{v_0^2 \sin 2\alpha}{g}$$

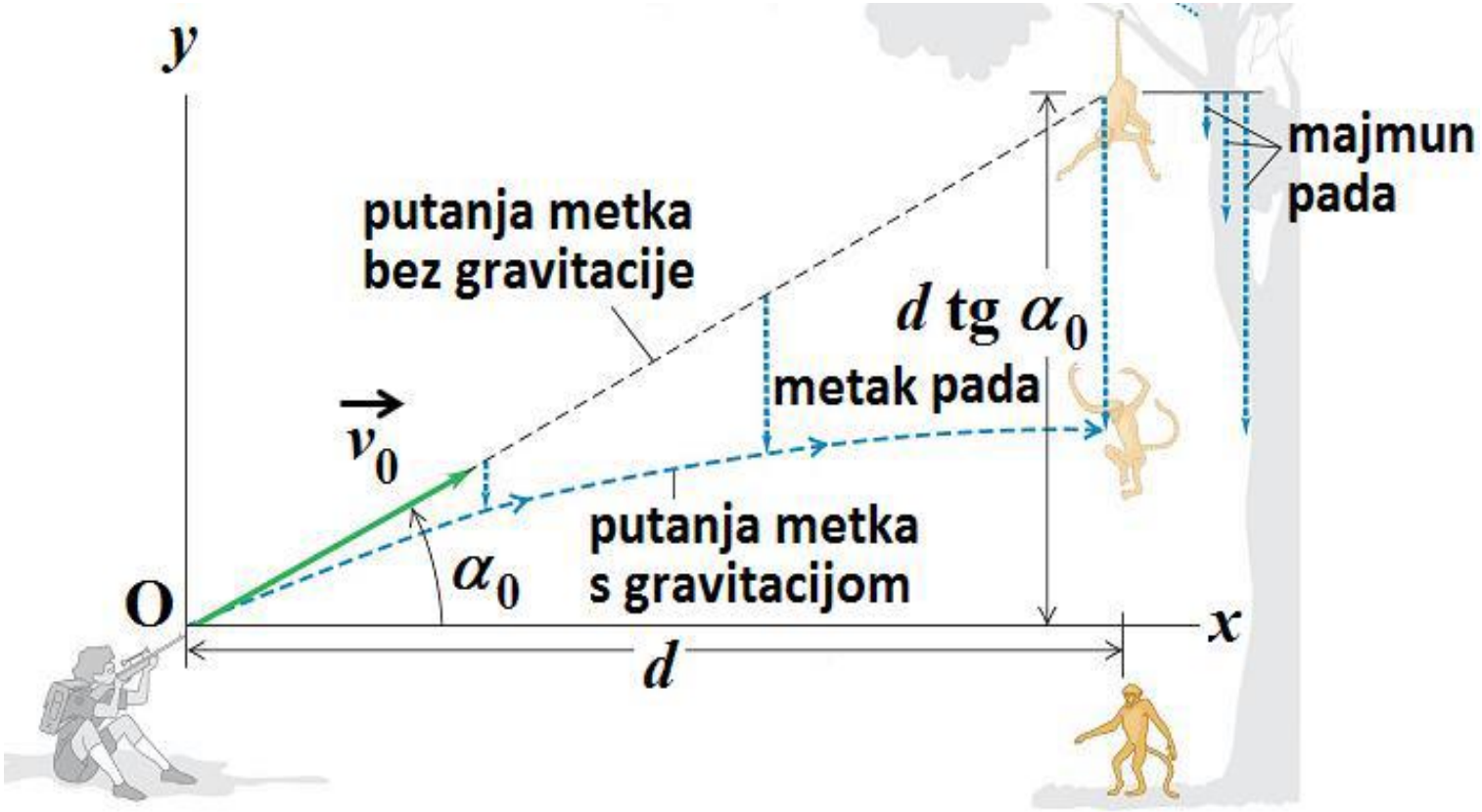
$$\frac{dD(\alpha_{max})}{d\alpha} = 0$$

$$\frac{2v_0^2 \cos 2\alpha_{max}}{g} = 0$$

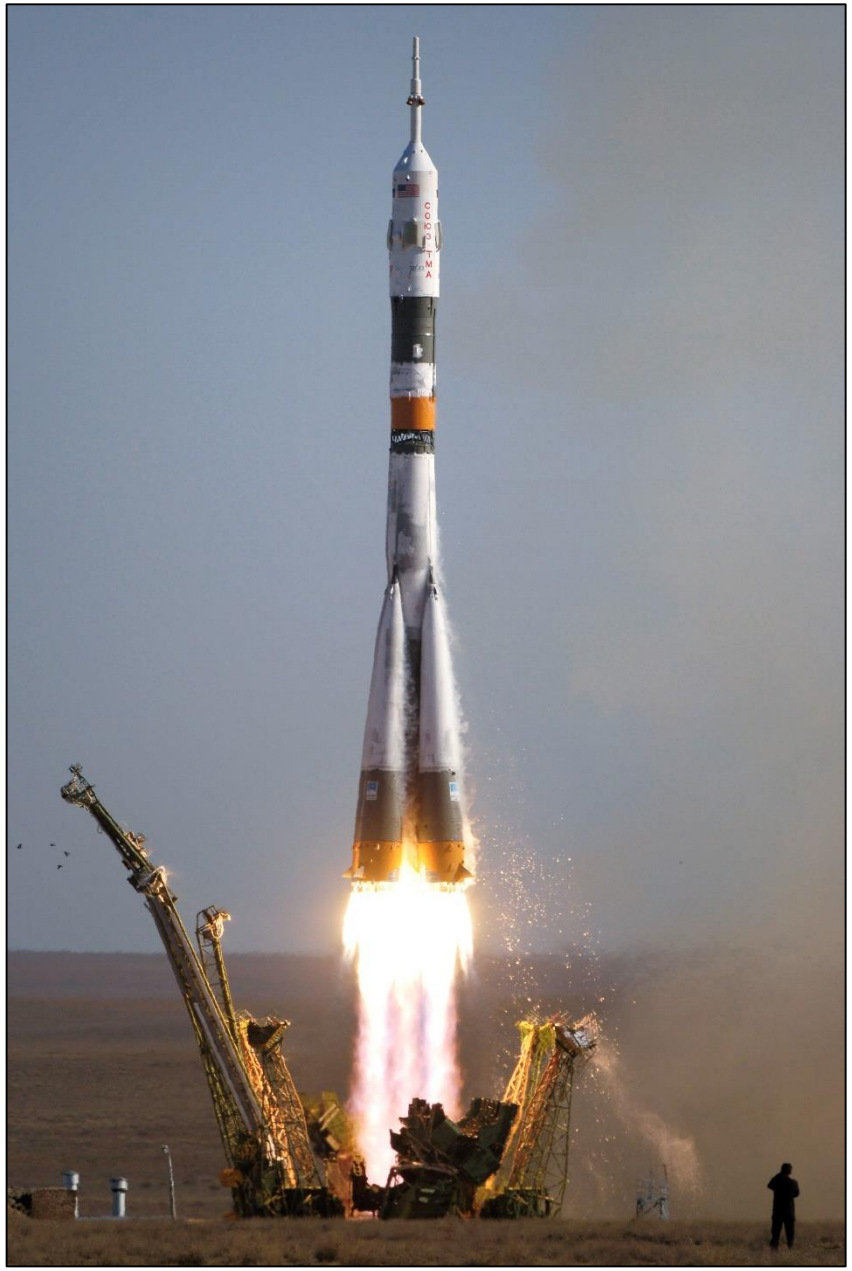
$$\alpha_{max} = 45°$$

$$D(\alpha) = D(90° - \alpha)$$

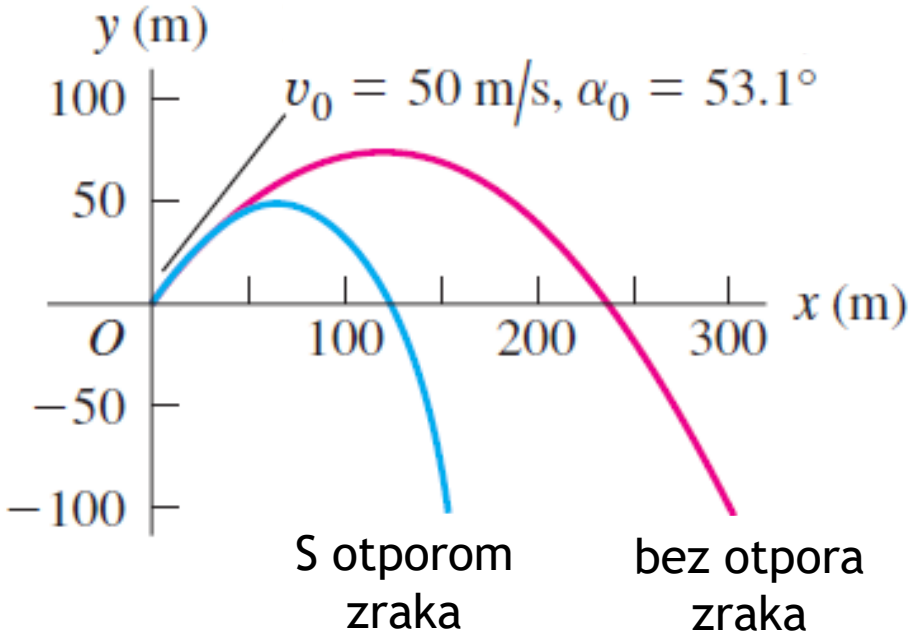
Kosi hitac



MIT Physics Demo - Monkey and a Gun



Balističke krivulje

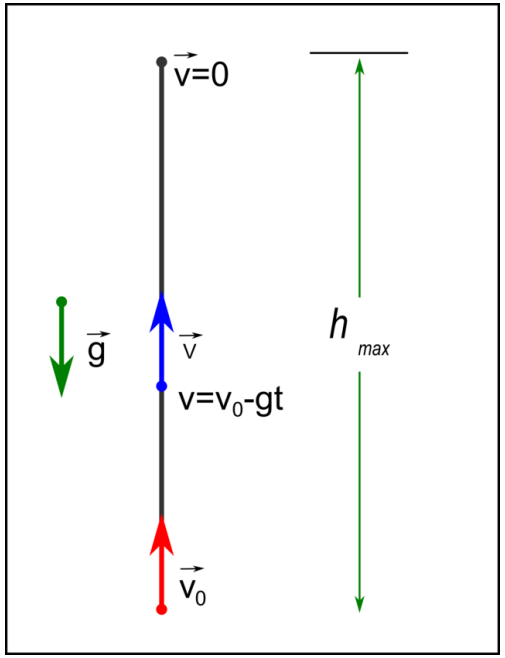


- Otpor zraka nije zanemariv.
- Putanja ovisi o obliku, presjeku, brzini projektila,
- temperaturi, tlaku, vjetru...

Kosi hitac

- Posebni slučajevi:
  - $\alpha = 90°$ vertikalni hitac prema gore
  - $\alpha = 270°$ vertikalni hitac prema dolje
  - $\alpha = 0°$ horizontalni hitac

Vertikalni hitac



Jednoliko usporeno pa jednoliko ubrzano gibanje (slobodni pad)

hitac u vis: $a = -g$

$\alpha = 90°$

$v = v_0 - gt$

$h = h_0 + v_0 t - \frac{1}{2} g t^2$

$v = 0$ tijelo se zaustavlja! $h = h_{max}$

$$v_0 - gt = 0 \quad \rightarrow \quad t = \frac{v_0}{g}$$

hitac prema dolje: $a = g$

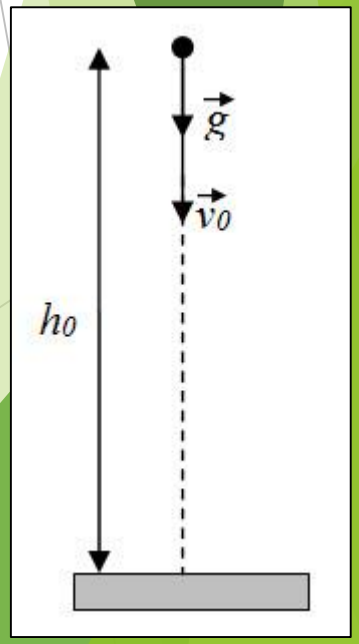
$\alpha = 270°$

$v = v_0 + gt$

$s = v_0 t + \frac{1}{2} g t^2$

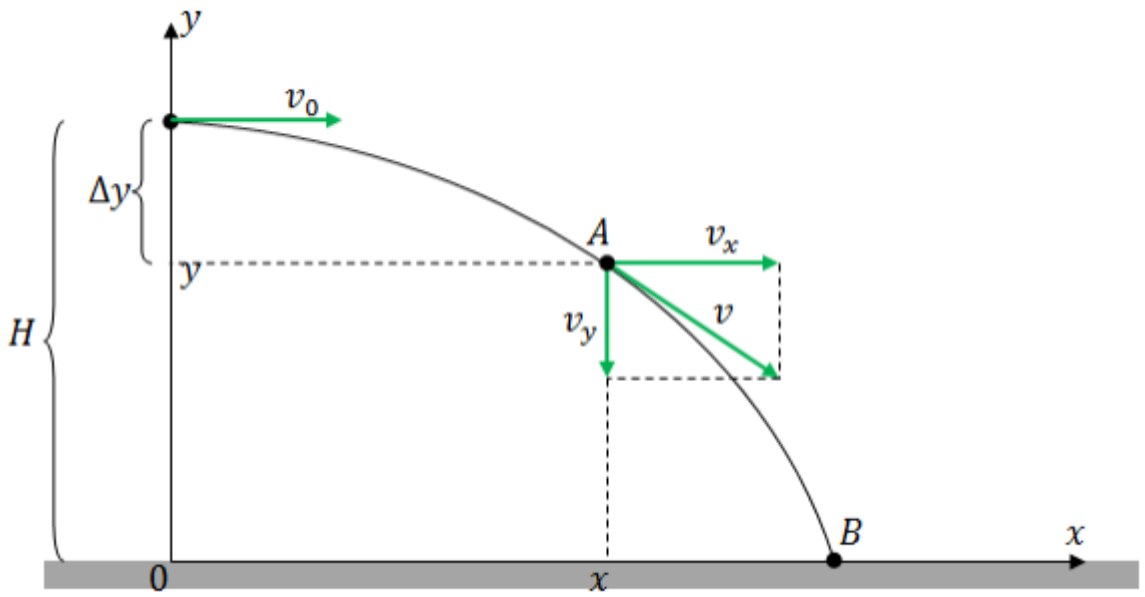
Ako je ishodište na tlu: $h = h_0 - v_0 t - \frac{1}{2} g t^2$

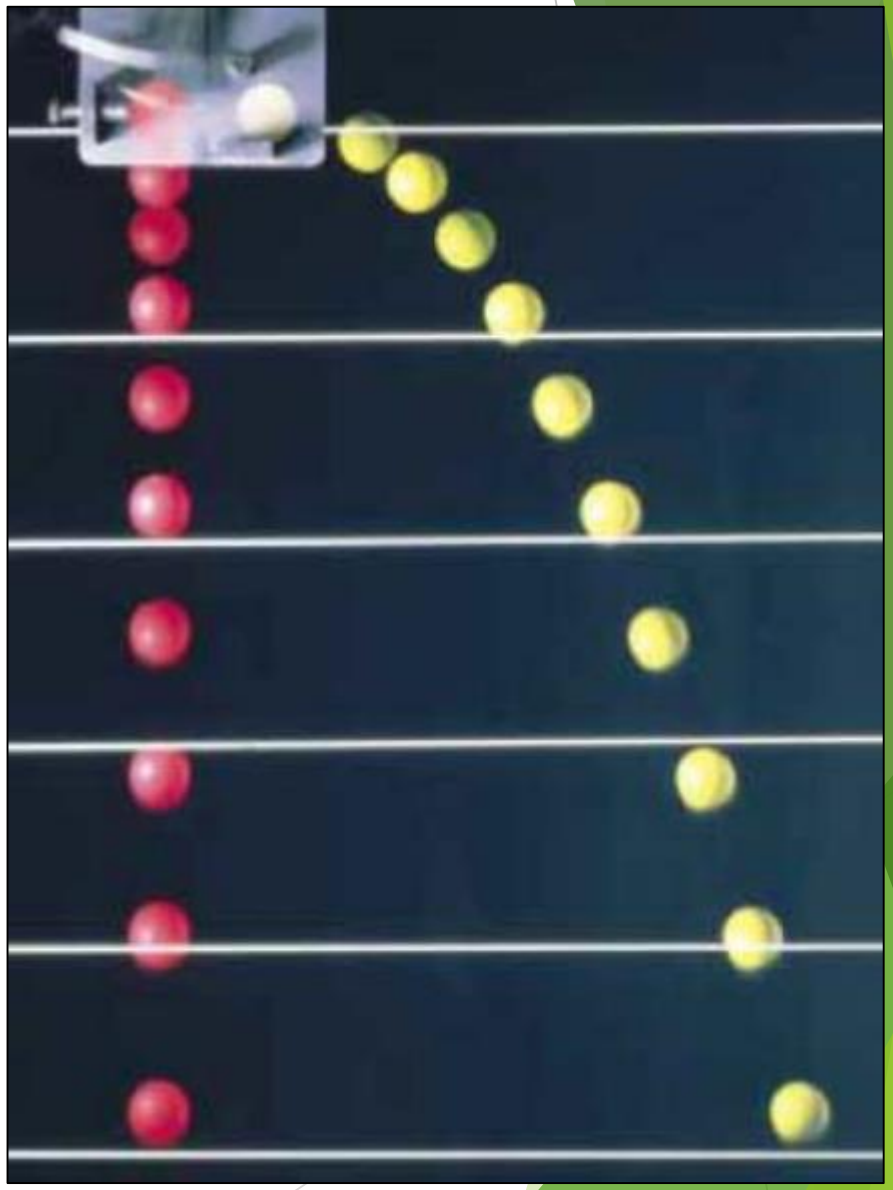
Jednoliko ubrzano gibanje

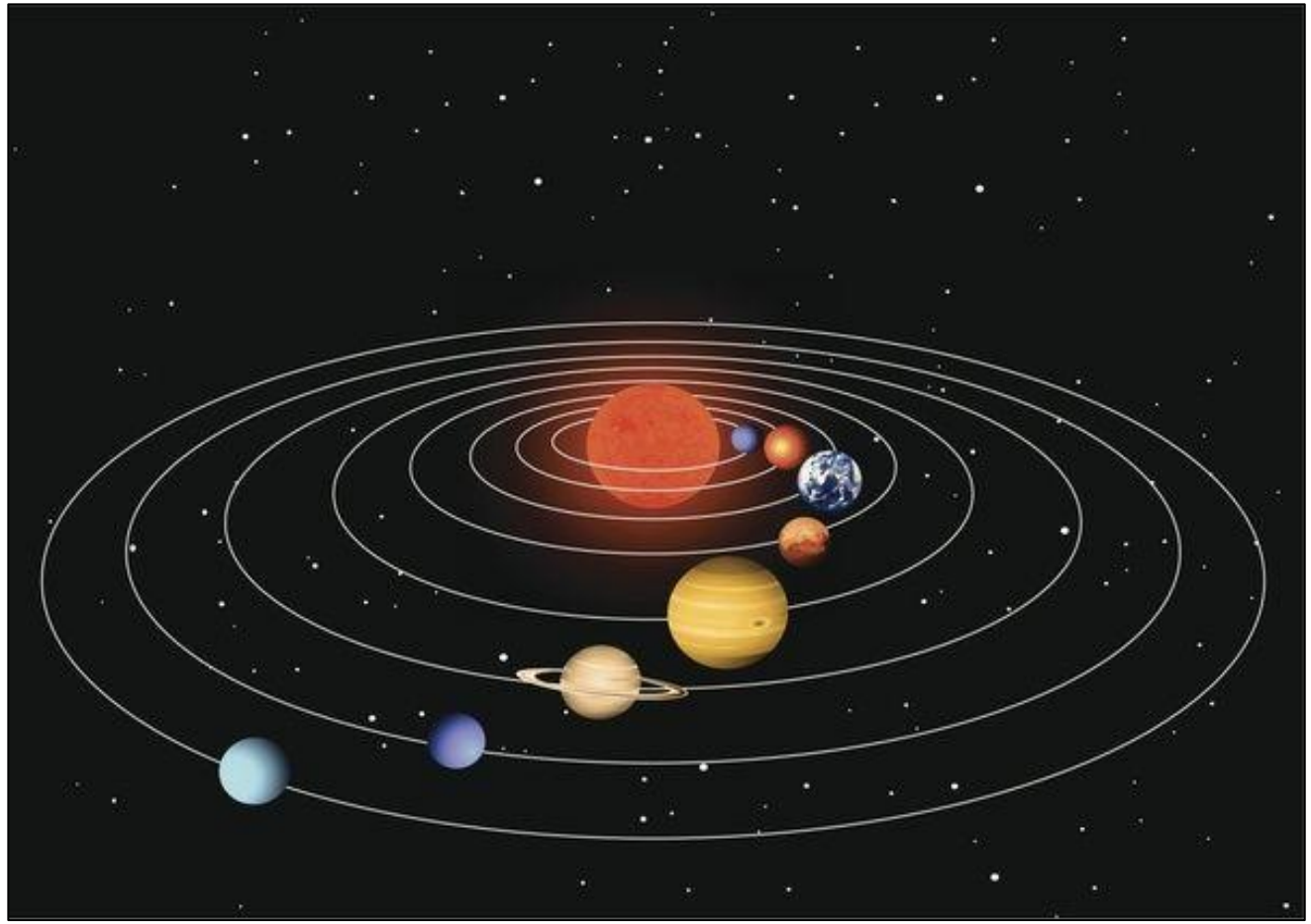


Horizontalni hitac

$\alpha = 0°$

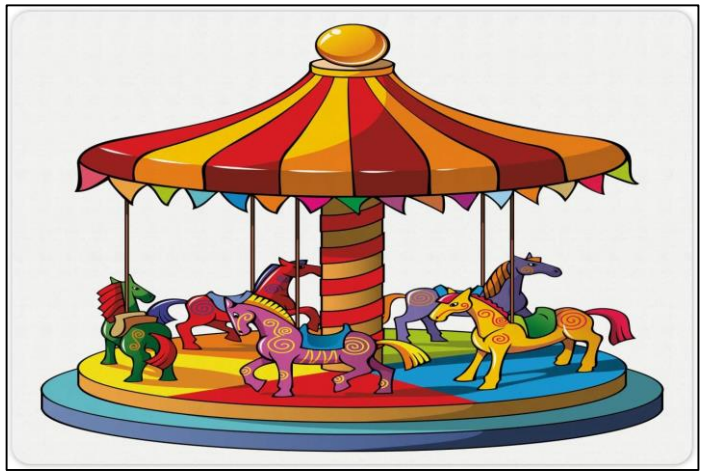


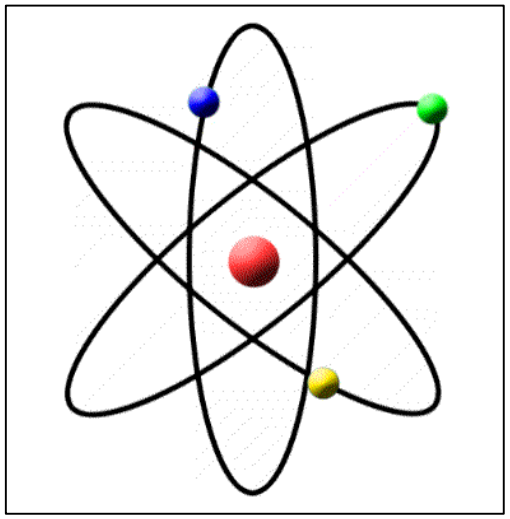




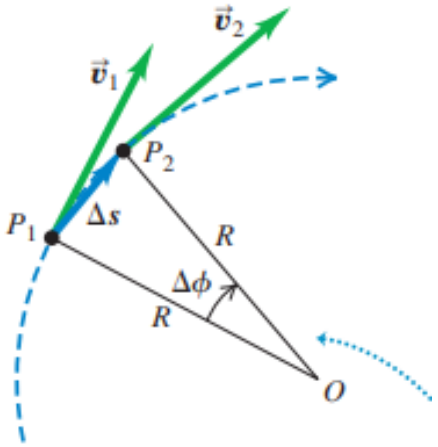
Jednoliko gibanje po kružnici

- najjednostavnije krivocrtno gibanje
  - Brzina je konstantna po iznosu, ali stalno mijenja smjer!





Jednoliko gibanje po kružnici



- Za vrijeme Δt materijalna točka prijeđe put Δs (dio kružnog luka), odnosno kut $\Delta\varphi$

$$\Delta s = r\Delta\varphi$$

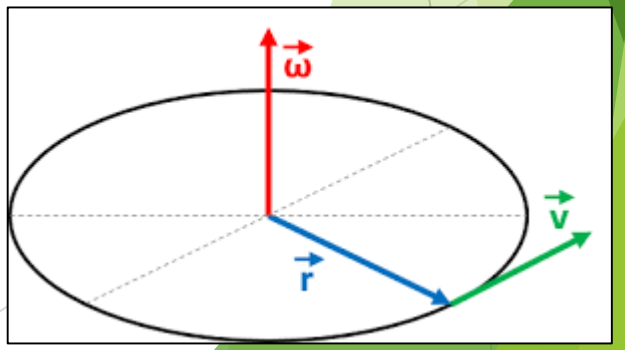
Linearna (obodna) brzina

$$v = \lim_{\Delta t \to 0} \frac{\Delta s}{\Delta t} = r \lim_{\Delta t \to 0} \frac{\Delta \varphi}{\Delta t} = r\frac{d\varphi}{dt} = r\omega$$

Kutna brzina

$$\omega = \frac{d\varphi}{dt} \quad [\mathrm{rad/s}]$$

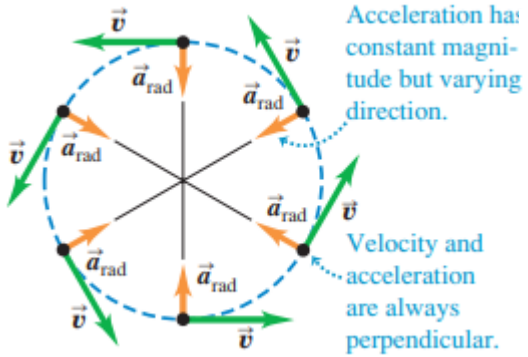
$$\vec{v} = \vec{\omega} \times \vec{r}$$



Jednoliko gibanje po kružnici

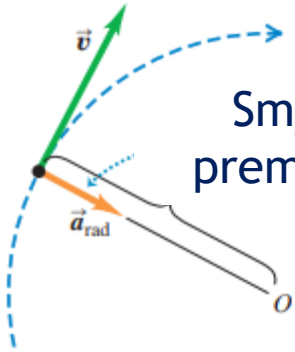
- Obodna brzina, iako ne mijenja iznos, mijenja smjer i zato je jednoliko kružno gibanje zapravo ubrzano gibanje.

Radijalna (centripetalna) akceleracija



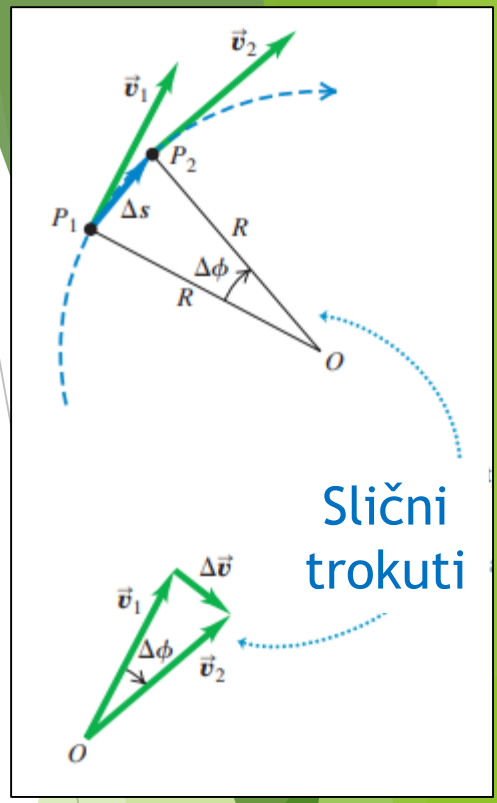
$$|\Delta\vec{v}| = v\Delta\varphi \quad /:\Delta t$$

$$a_r = \lim_{\Delta t\to 0}\frac{|\Delta\vec{v}|}{\Delta t} = \lim_{\Delta t\to 0}\frac{v\Delta\varphi}{\Delta t} = v\lim_{\Delta t\to 0}\frac{\Delta\varphi}{\Delta t} = v\omega$$



Smjer akceleracije – prema središtu kružnice

$$\vec{a}_r = -r\omega^2\vec{r}_0 = -\frac{v^2}{r}\vec{r}_0 = \vec{\omega}\times\vec{v}$$



$\vec{a}_r$ - stalni iznos, promjenjivi smjer

$\vec{v} \perp \vec{a}_r$

Jednoliko gibanje po kružnici

- Kruženje konstantnom kutnom brzinom:

$$\frac{d\varphi}{dt} = \omega = konst.$$

- Integriranjem dobijemo put (kut)

$$\varphi = \omega t + \varphi_0$$

Uvodimo:

Ophodno vrijeme, T = vrijeme potrebno da materijalna točka jednom obiđe kružnicu.

Frekvencija, f = broj okreta u sekundi.

$$\omega = 2\pi f; \quad f = \frac{1}{T}$$

Nejednoliko kružno gibanje

Mijenja se i iznos obodne brzine s vremenom

→ imamo i tangencijalnu akceleraciju:

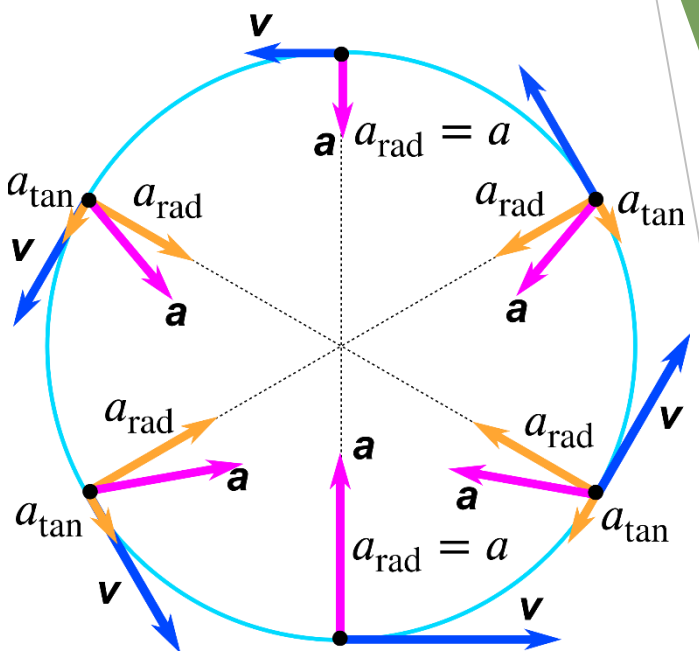
$$a_t = \frac{dv}{dt} = \frac{d(r\omega)}{dt} = r\frac{d\omega}{dt} = r\alpha$$

Kutna akceleracija:

$$\alpha = \lim_{\Delta t \to 0} \frac{\Delta\omega}{\Delta t} = \frac{d\omega}{dt} = \frac{d^2\varphi}{dt^2}$$

Ukupna akceleracija:

$$\vec{a} = \vec{a}_t + \vec{a}_r; \quad a = \sqrt{(a_t^2 + a_r^2)}$$



Poseban slučaj $\underline{\alpha = konst.}$

Integriranje $d\omega = \alpha dt \quad \rightarrow \quad \omega = \alpha t + \omega_0$

$$d\varphi = (\alpha t + \omega_0)dt$$

Integriranjem $\rightarrow \quad \varphi = \frac{\alpha}{2}t^2 + \omega_0 t$

Usporedba pravocrtnog i kružnog gibanja

Pravocrtno gibanje	Kružno gibanje
$a = \frac{dv}{dt} = \frac{d^2s}{dt^2}$	$\alpha = \frac{d\omega}{dt} = \frac{d^2\varphi}{dt^2}$
$v = \frac{ds}{dt}$	$\omega = \frac{d\varphi}{dt}$
$v = v_0 + at$	$\omega = \omega_0 + \alpha t$
$v^2 = 2as + v_0^2$	$\omega^2 = 2\alpha\varphi + \omega_0^2$
$s = s_0 + vt$	$\varphi = \varphi_0 + \omega t$
$s = s_0 + v_0 t + \frac{1}{2}at^2$	$\varphi = \varphi_0 + \omega_0 t + \frac{1}{2}\alpha t^2$