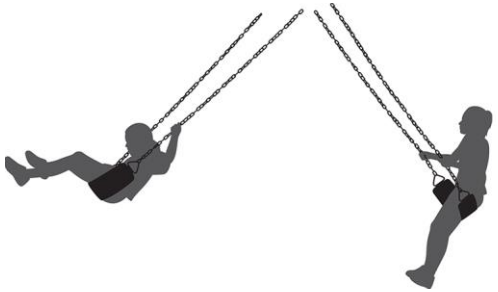
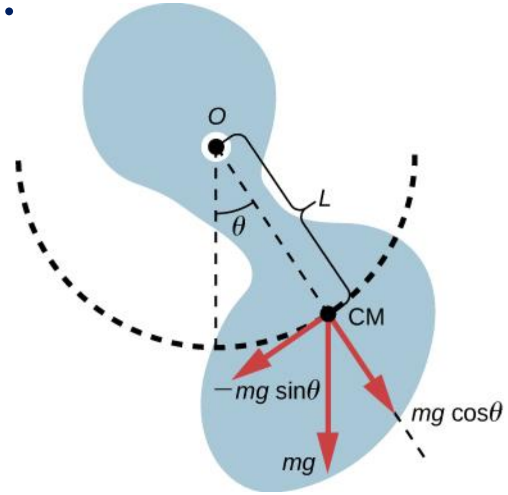
Fizika I

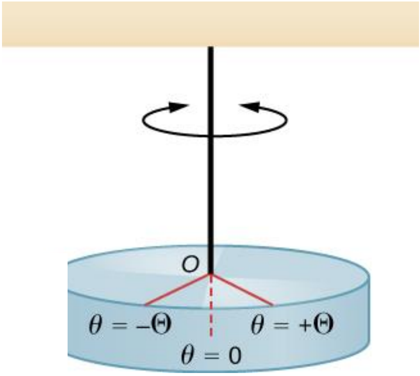
TITRANJE

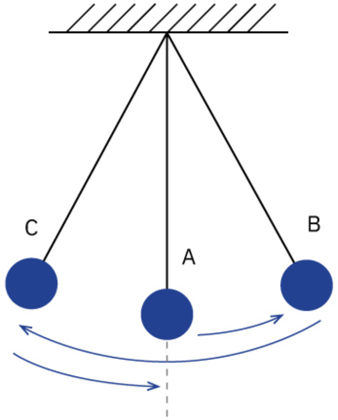
TITRANJE

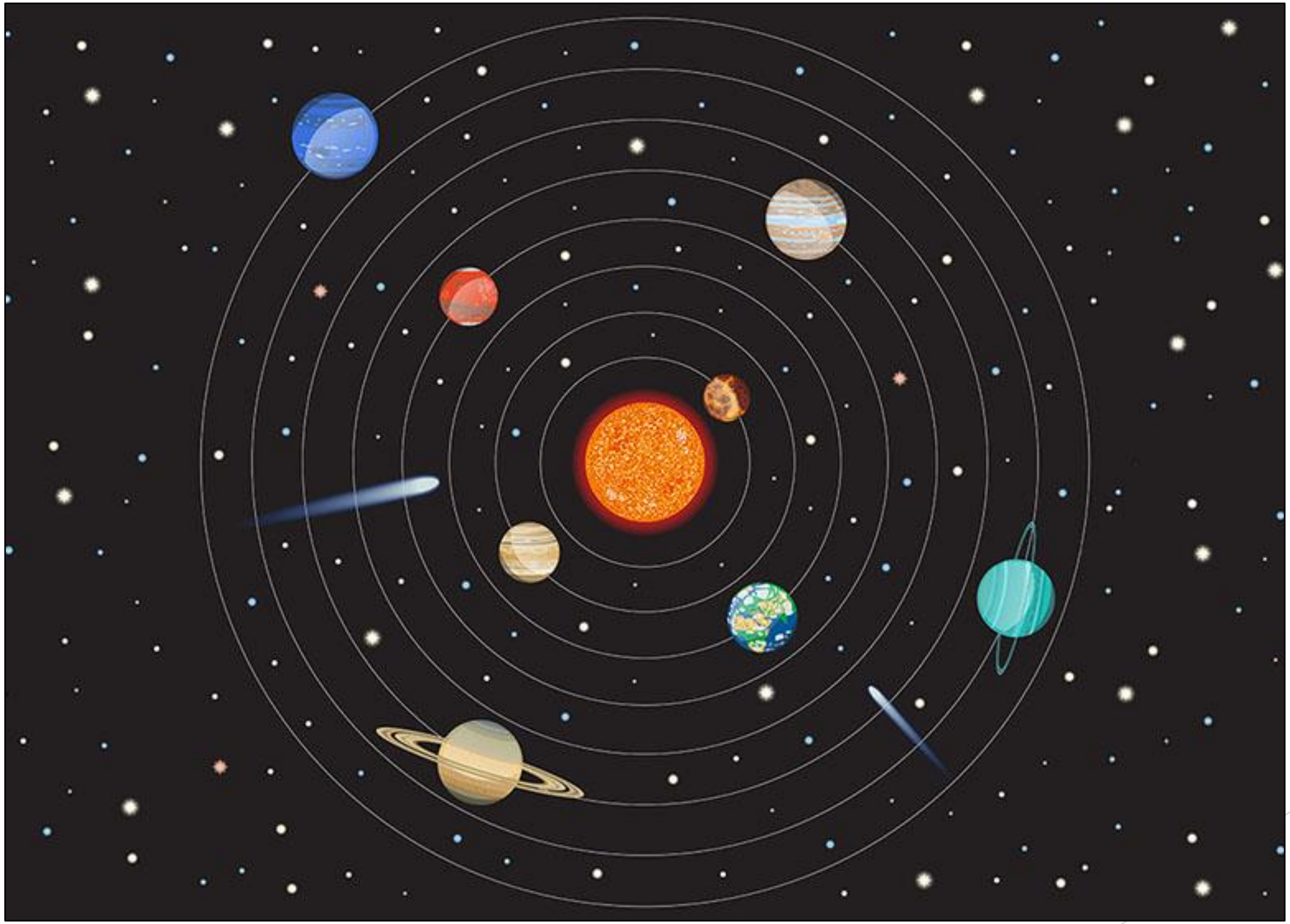
- Titranje (oscilatorno gibanje) – gibanje pri kojem materijalna čestica prevaljuje određeni put između dvaju krajnjih položaja, vraća se istim putem nakon čega se to gibanje periodički ponavlja.
- Čestica koja titra – oscilator.

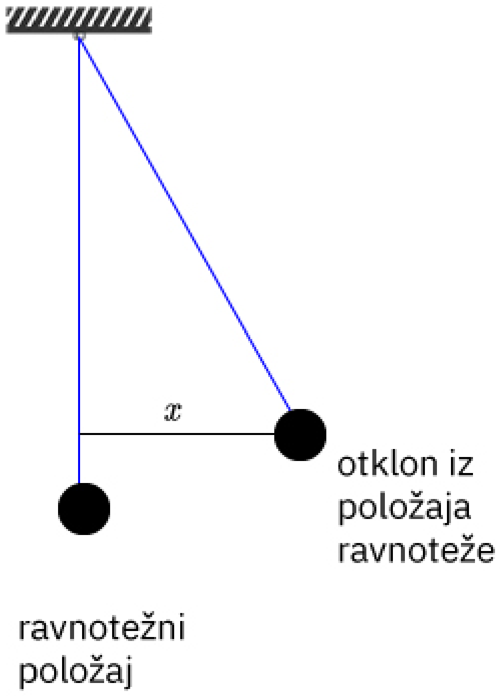


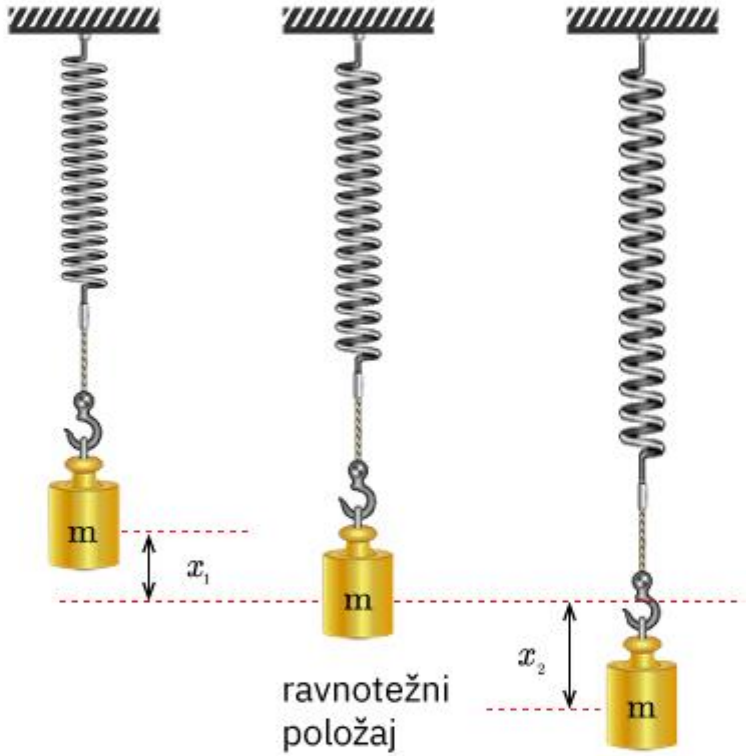






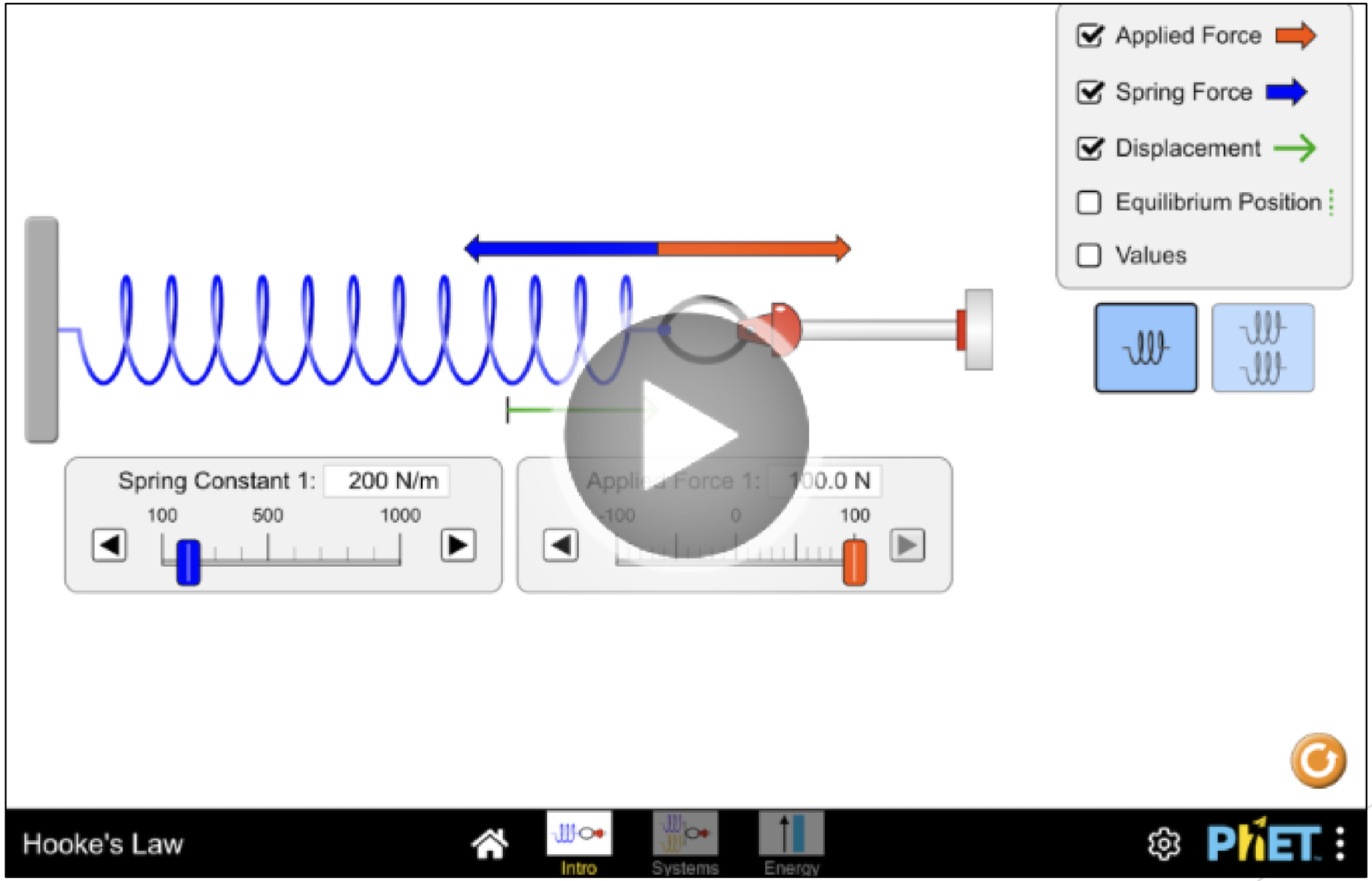






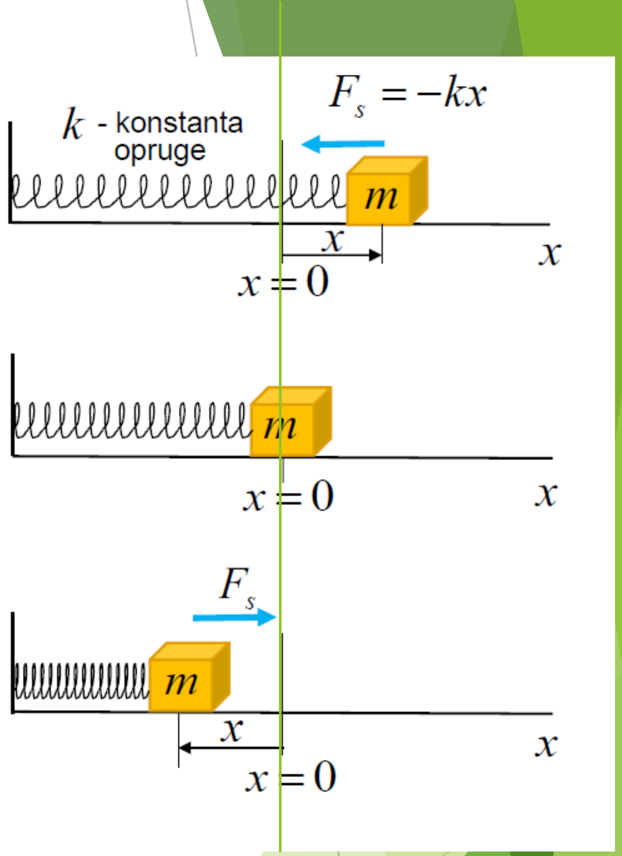
x_1, x_2 - elongacija, odn. pomak iz ravnotežnog položaja

Simulacija: Hookeov zakon



Harmoničko titranje

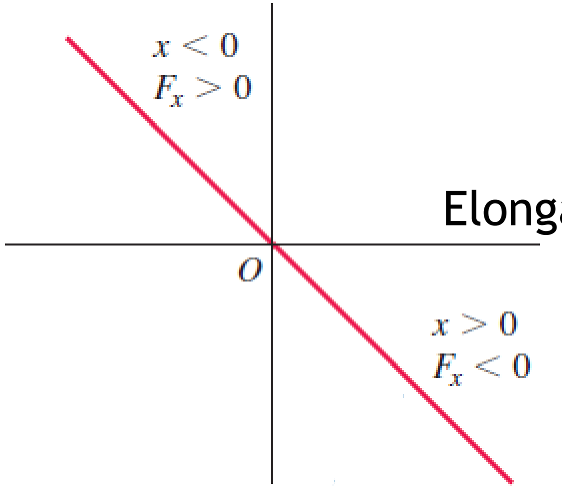
- Posebno pravilan način titranja.
- Čestica koja harmonički titra – harmonički oscilator.
- Uzrok gibanju je elastična (harmonička) sila.
- Elastična sila je povratna sila – uvijek suprotna od pomaka čestice iz ravnotežnog položaja.
- Povratna sila je direktno proporcionalna elongaciji iz ravnoteže x (s): (Hookeov zakon).



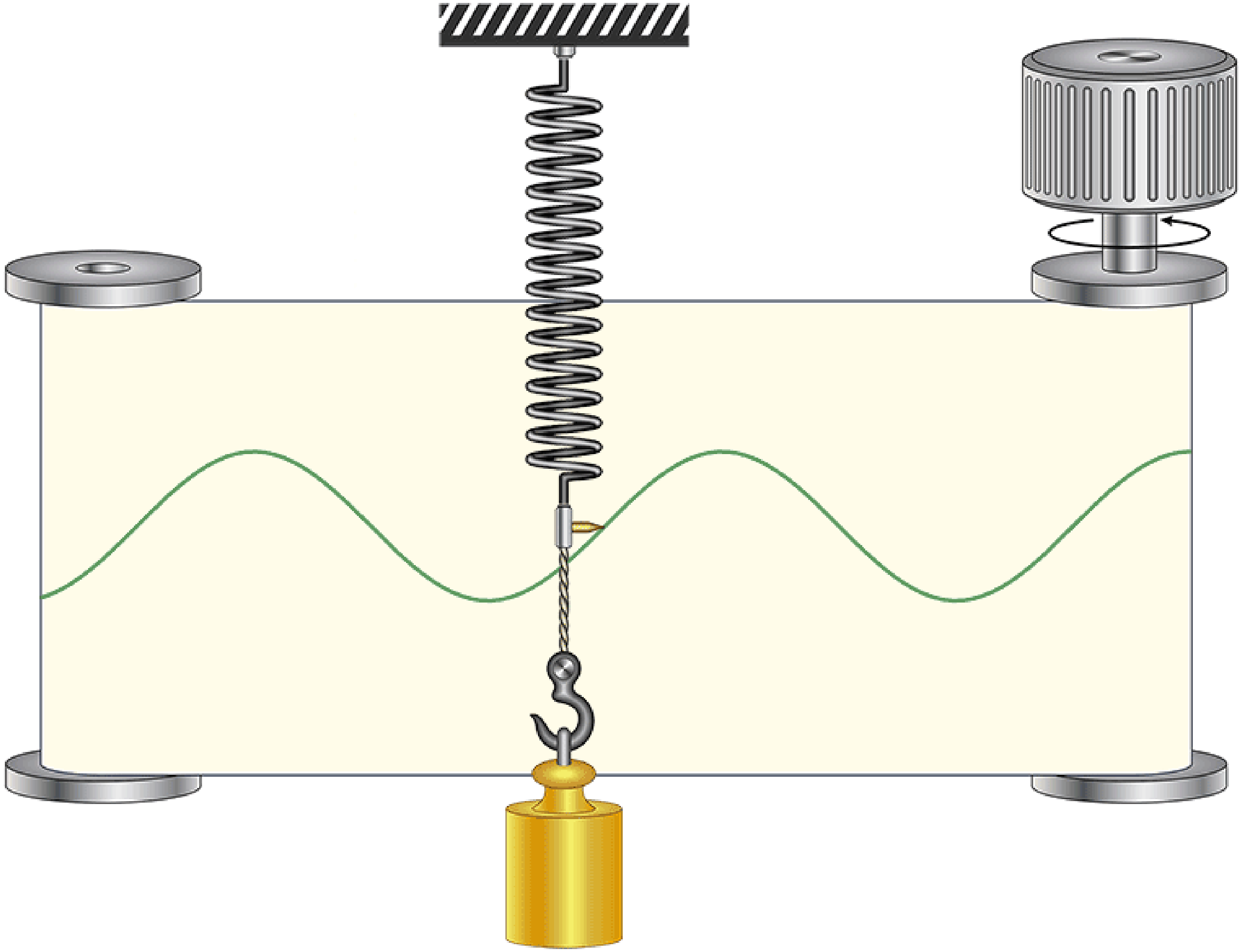
ravnotežni položaj

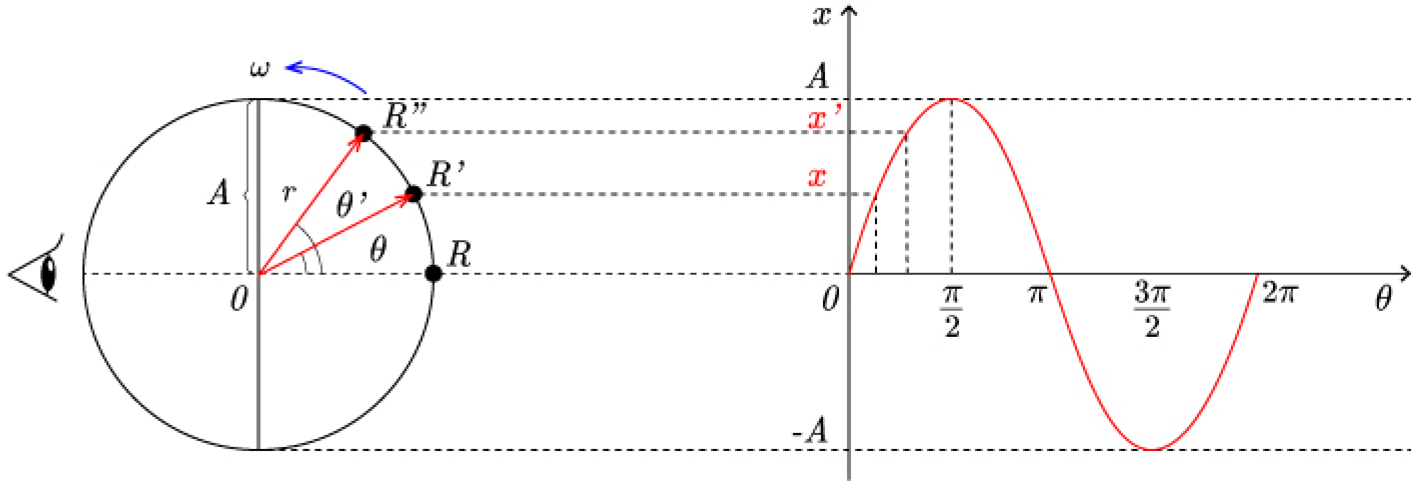
$$F = -kx$$

Povratna sila, F



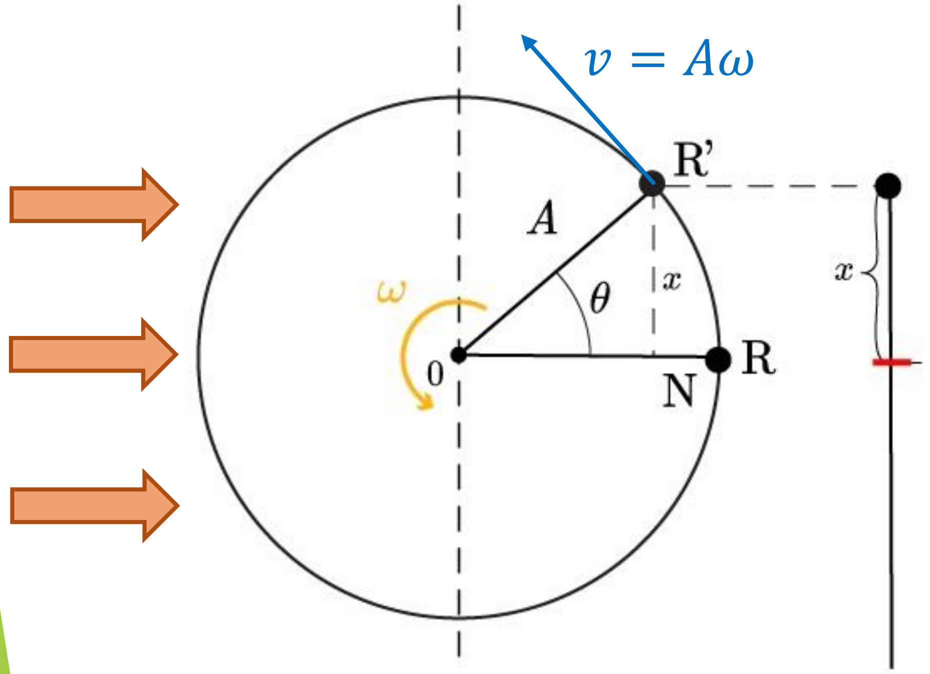
Elongacija, x





ω
A
r
θ'
θ
R"
R'
R
0
x
A
x'
x
0
π/2
π
3π/2
2π
θ
-A

Harmoničko titranje kao projekcija jednolikog gibanja po kružnici



- Čestica mase m giba se jednoliko kutnom brzinom $\omega = \theta / t$ u vertikalnoj ravnini po kružnici polumjera A.
- Projekcija čestice (oscilator) titra gore – dolje

$$x = Asin\omega t$$

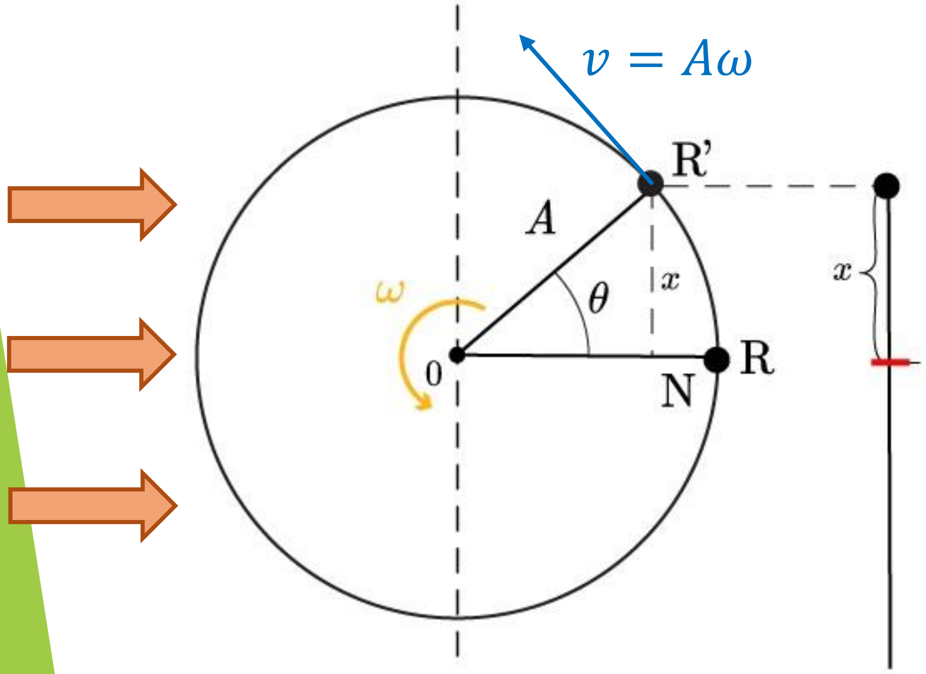
$$v = A\omega \cos \omega t$$

$$a = -\frac{v^2}{A} \sin \omega t$$

$$F = -m\frac{v^2}{A} \sin \omega t = -m\omega^2 x$$

- Povratna harmonička sila: $F = -kx$.

Harmoničko titranje kao projekcija jednolikog gibanja po kružnici



$$k = m\omega^2$$

- Kružna frekvencija: $\omega = \sqrt{\frac{k}{m}} = 2\pi f$
- Period: $T = 2\pi\sqrt{\frac{m}{k}}$
- Frekvencija: $\nu = f = \frac{1}{T} = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$

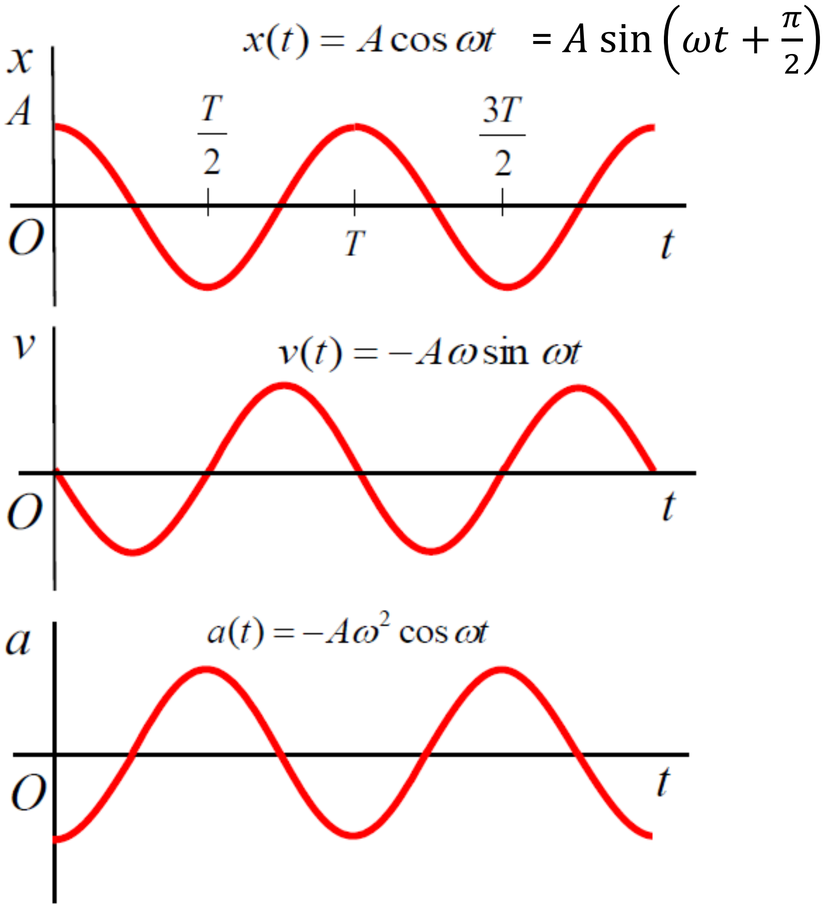
$$x = s = A sin(\omega t + \varphi_0)$$

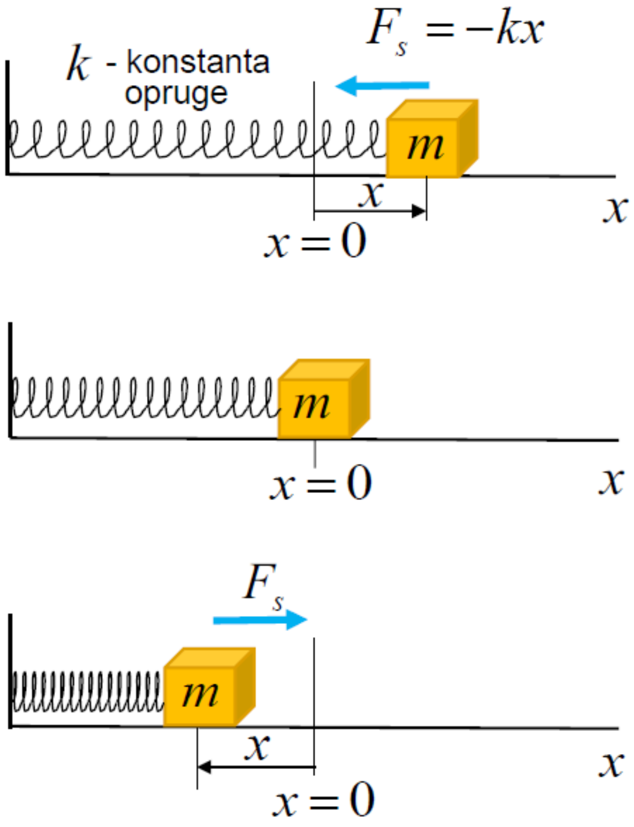
elongacija

početna faza

amplituda

Jednadžba gibanja harmoničkog oscilatora





Hookeov zakon:

$$F = -ks$$

Drugi Newtonov zakon:

$$ma = -ks$$

$$m\frac{\mathrm{d}^2 s}{\mathrm{d}t^2} = -ks$$

Diferencijalna jednadžba:

$$\frac{\mathrm{d}^2 s}{\mathrm{d}t^2} + \frac{k}{m}s = 0$$

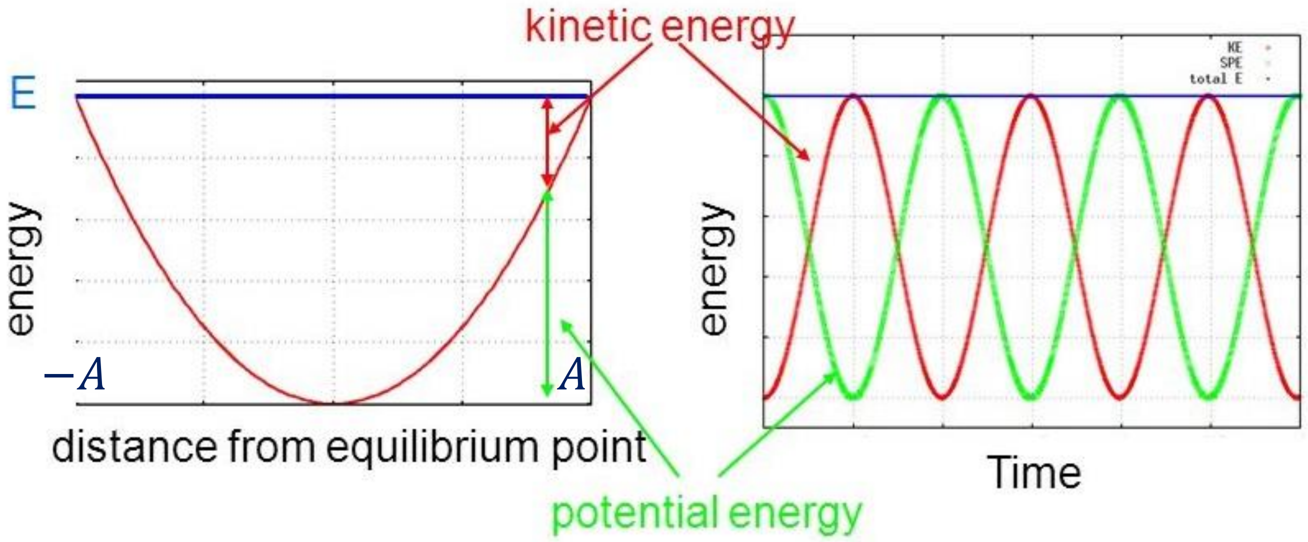
ω^2

Rješenje:

$$s = A\sin(\omega t + \varphi_0)$$

Energija harmoničkog oscilatora

- Simulacija (Geogebra)



Potencijalna energija:

$$E_p = \frac{1}{2}kx^2$$

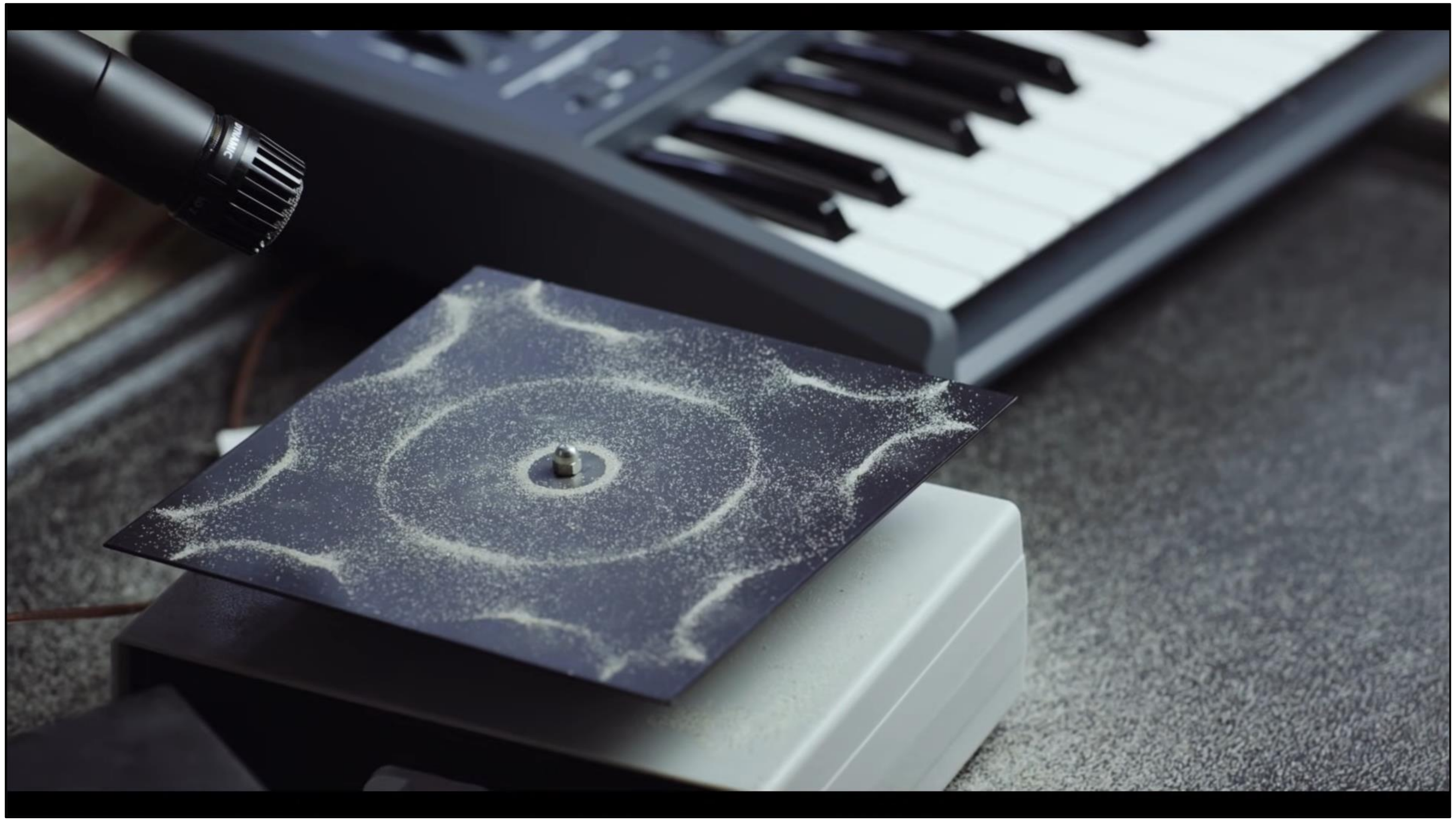
Kinetička energija:

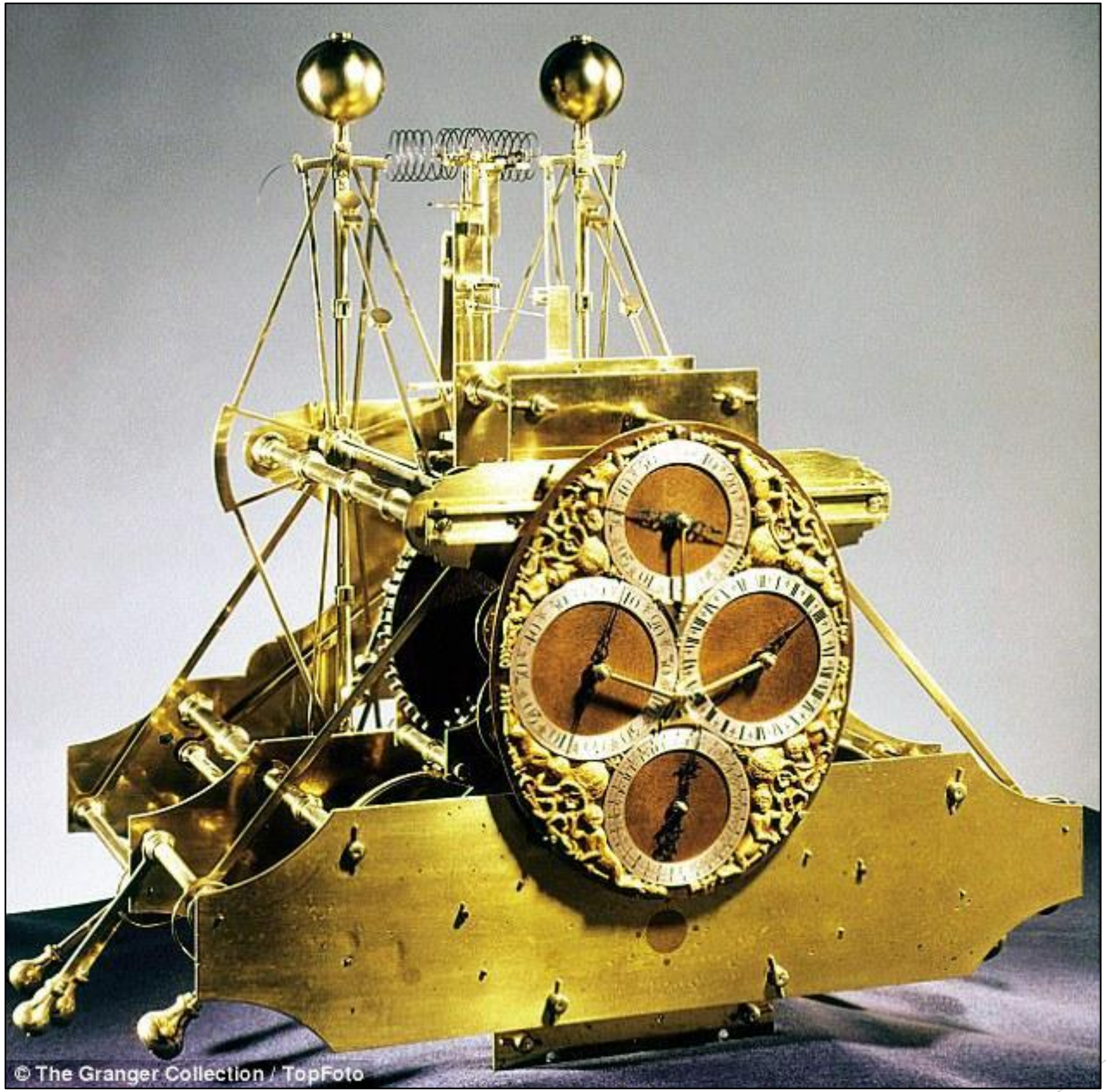
$$E_k = \frac{1}{2}mv^2$$

Ukupna energija

$$E = E_k + E_p$$

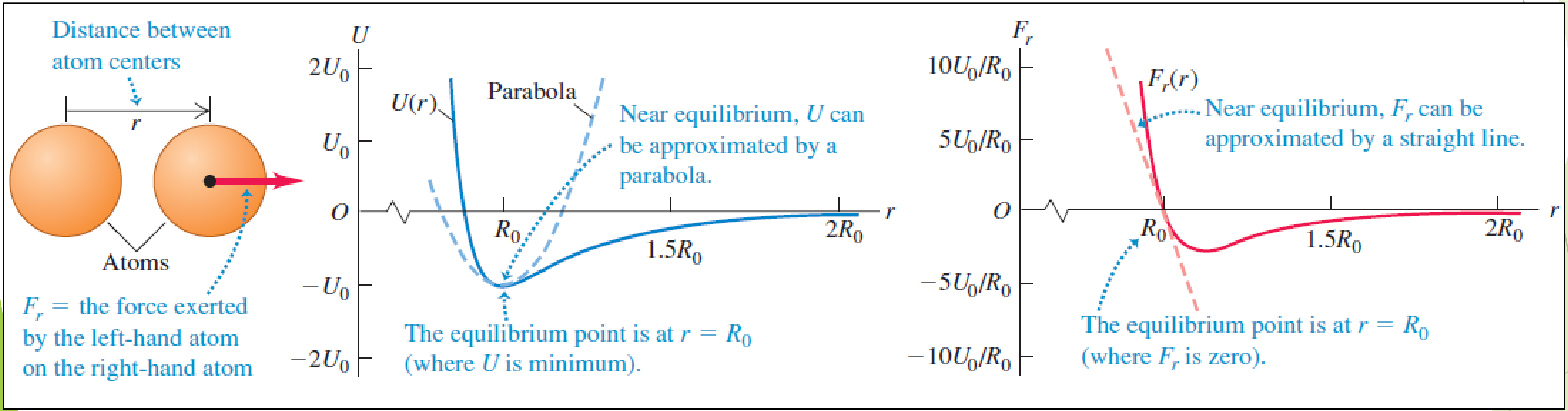
$$E = \frac{1}{2}kA^2$$





Vibracije dvoatomne molekule - primjer

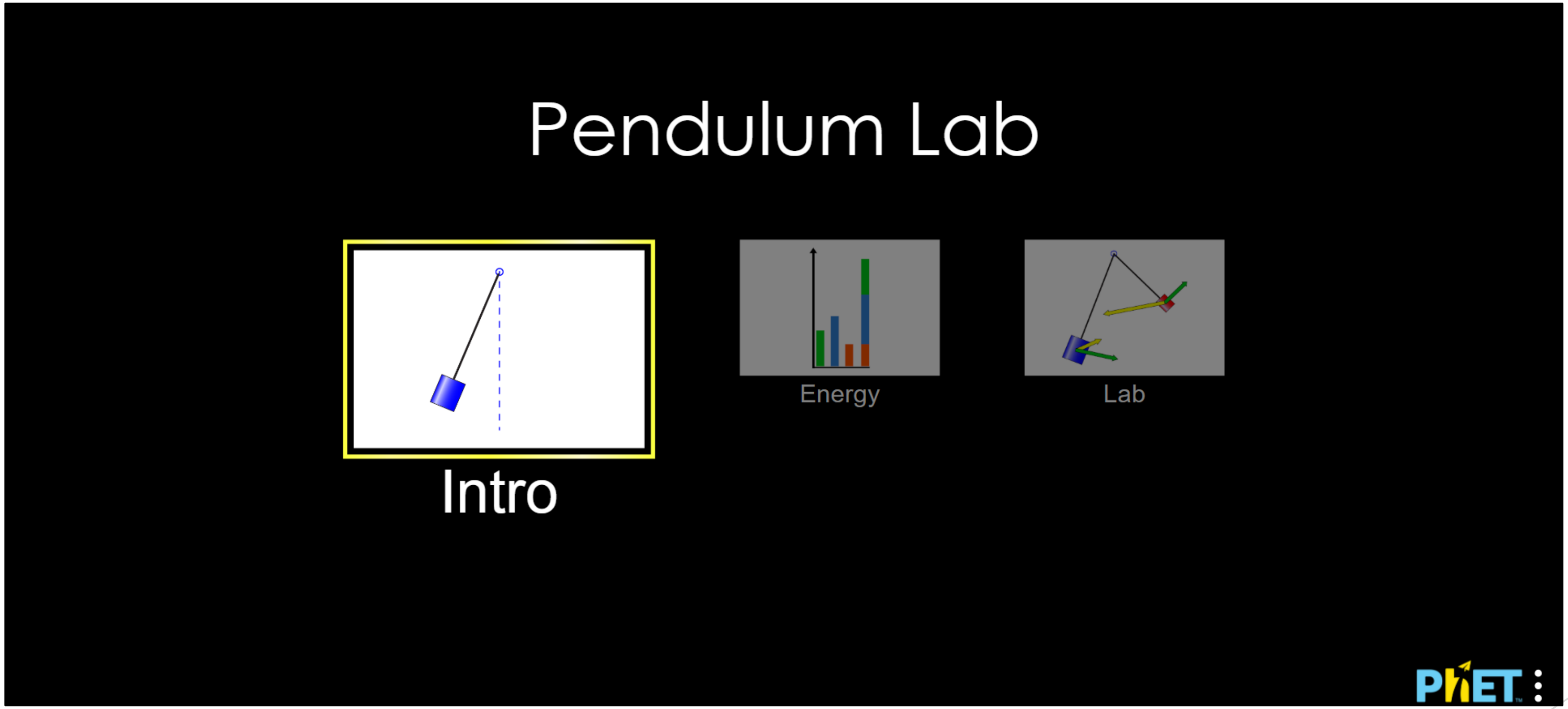
- U stvarnosti su mnoga gibanja samo približno harmonička.
- Dvoatomna molekula – anharmonička sila.



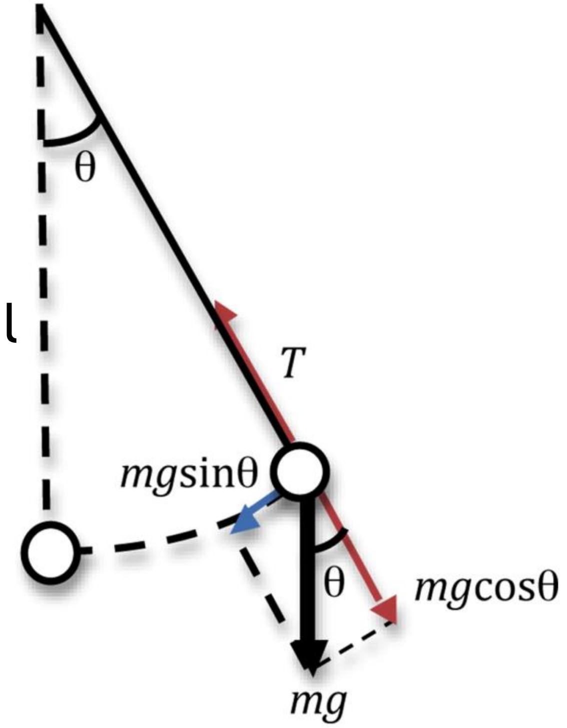
$$U = U_0\left[\left(\frac{R_0}{r}\right)^{12} - 2\left(\frac{R_0}{r}\right)^{6}\right]$$

van der Waalsova interakcija

Simulacija: Matematičko njihalo



Matematičko njihalo



- Idealan sustav: materijalna točka mase m obješena o nerastezljivu nit zanemarive mase i duljine l.
- Dobra aproksimacija: kuglica obješena na kraj duge, tanke nerastezljive niti.
- Kuglica se zbog napete niti giba po kružnici polumjera l.

Newtonov zakon:

$$M = I\alpha = I\frac{\mathrm{d}^2\theta}{\mathrm{d}t^2}$$

$$I = ml^2$$

$$F_t = mg\sin\theta$$

$$M = -mgl\sin\theta$$

$$\frac{\mathrm{d}^2\theta}{\mathrm{d}t^2} + \frac{g}{l}\sin\theta = 0$$

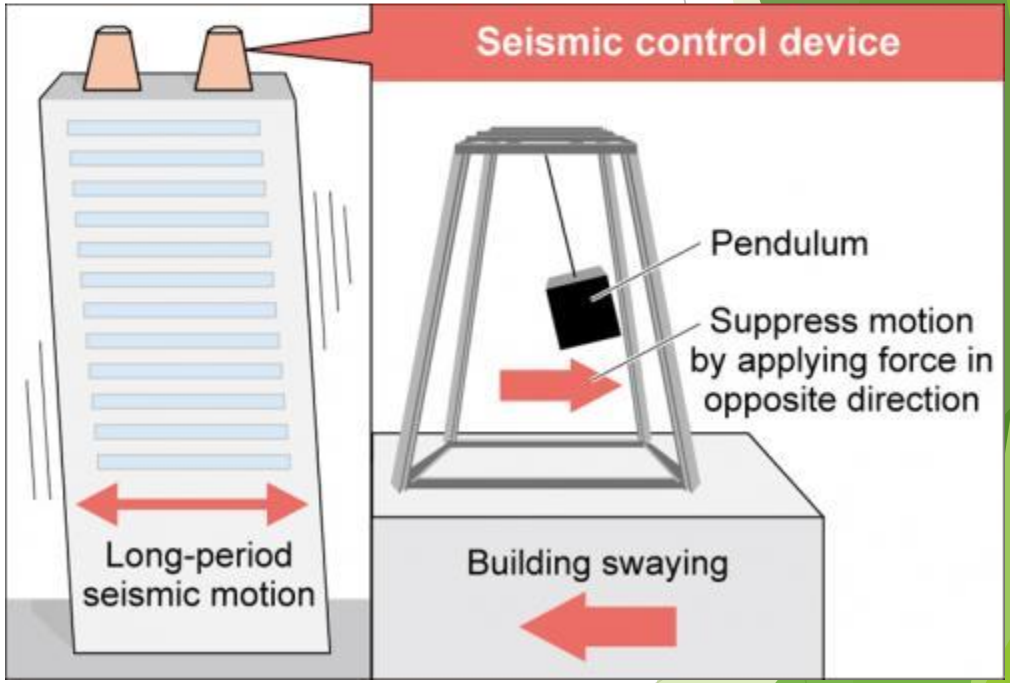
$$\theta \ll \quad \rightarrow \quad \sin\theta \cong \theta$$

Mali kut u radijanima:

$$\frac{\mathrm{d}^2\theta}{\mathrm{d}t^2} + \underbrace{\frac{g}{l}}_{\omega^2}\theta = 0$$

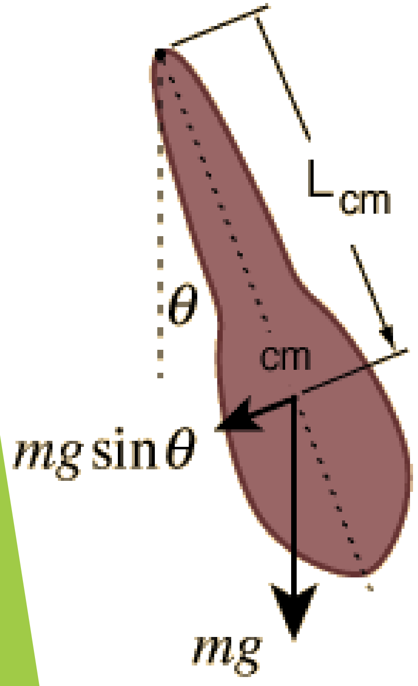
$$T = 2\pi\sqrt{\frac{l}{g}}$$

$$\theta = \theta_0\sin(\omega t + \varphi_0)$$



Seismic control device
Pendulum
Suppress motion
by applying force in
opposite direction
Long-period
seismic motion
Building swaying

Fizičko njihalo



- Kruto tijelo obješeno oko čvrste osi koja ne prolazi njegovim težištem.
- Spojnica između objesišta i težišta (cm) (duljine L) zatvara s težinom $\vec{G}$ kut θ.

Newtonov zakon:

$$M = I\alpha = I\frac{\mathrm{d}^2\theta}{\mathrm{d}t^2}$$

$$M = -Lmg\sin\theta$$

$$\frac{\mathrm{d}^2\theta}{\mathrm{d}t^2} + \frac{Lmg}{I}\sin\theta = 0$$

$$\theta \ll \quad \rightarrow \quad \sin\theta \cong \theta$$

$$\frac{\mathrm{d}^2\theta}{\mathrm{d}t^2} + \frac{Lmg}{I}\theta = 0$$

$$T = 2\pi\sqrt{\frac{I}{Lmg}}$$

ω^2

$$\theta = \theta_0\sin(\omega t + \varphi_0)$$

Prigušeno titranje

U stvarnim sustavima imamo otpor sredstva u kojem tijelo titra:

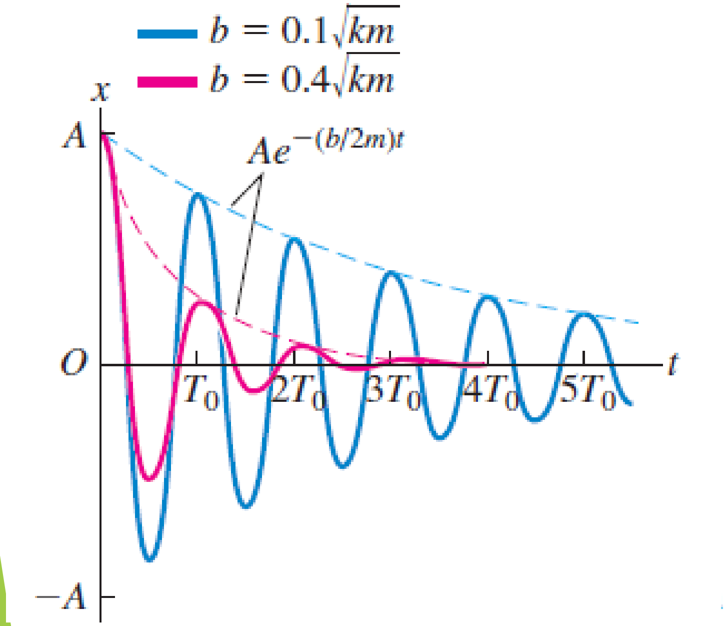
- Oscilator titra sve manjom amplitudom sve dok titranje potpuno ne utrne.
- Energija se gubi u okolinu.

- Najjednostavnije, otpor ovisi o brzini: $F_0 = -bv \qquad (v = \mathrm{d}s/\mathrm{d}t)$.
- Jednadžba gibanja: $m\frac{\mathrm{d}^2 s}{\mathrm{d}t^2} + b\frac{\mathrm{d}s}{\mathrm{d}t} + ks = 0$
  - $2\gamma = \frac{b}{m}$ (jakost sile otpora sredstva)
  - $\omega_0 = \sqrt{\frac{k}{m}}$ (vlastita kružna frekvencija)

$$\frac{\mathrm{d}^2 s}{\mathrm{d}t^2} + 2\gamma\frac{\mathrm{d}s}{\mathrm{d}t} + {\omega_0}^2 s = 0$$

Prigušeno titranje

$$\frac{d^2s}{dt^2} + 2\gamma\frac{ds}{dt} + {\omega_0}^2 s = 0 \qquad rj.: \qquad s = A_0 e^{-\gamma t} sin(\omega t + \varphi_0)$$



- $A = A_0 e^{-\gamma t}$ amplituda se eksponencijalno smanjuje s vremenom.
- $\omega = \sqrt{{\omega_0}^2 - \gamma^2}$ frekvencija nešto manja od vlastite, ω_0 $\rightarrow$ period T veći od T_0.

Prisilne oscilacije

- Izgubljena energija prigušenog titranja može se nadoknaditi vanjskom periodičkom silom:

$$F = F_0 \cos \omega_f t$$

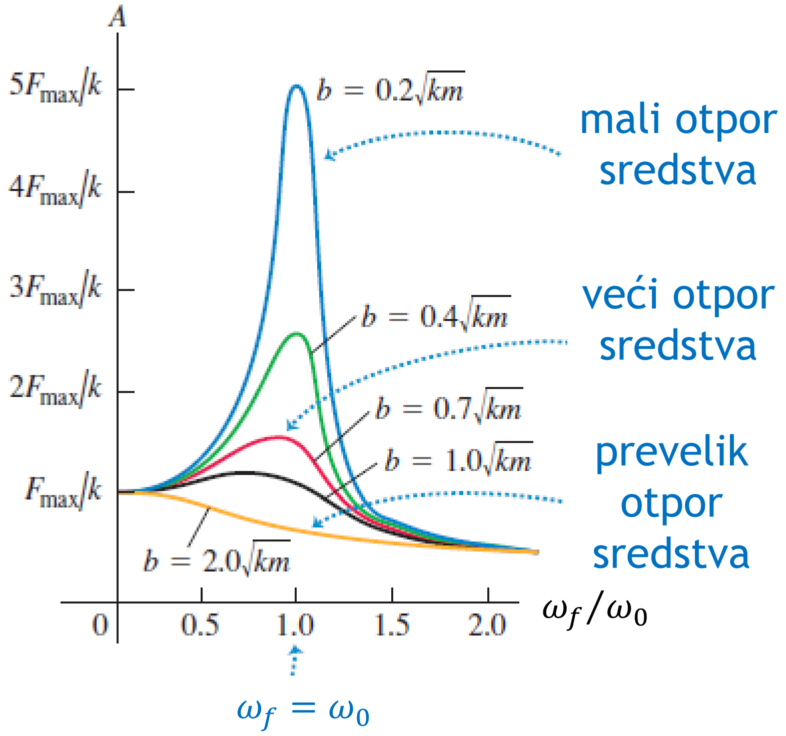
- Jednadžba gibanja: $m\frac{\mathrm{d}^2 s}{\mathrm{d}t^2} + b\frac{\mathrm{d}s}{\mathrm{d}t} + ks = F_0 \cos \omega_f t$
- Sustav titra nametnutom frekvencijom: $s = Asin(\omega_f t + \varphi_f)$

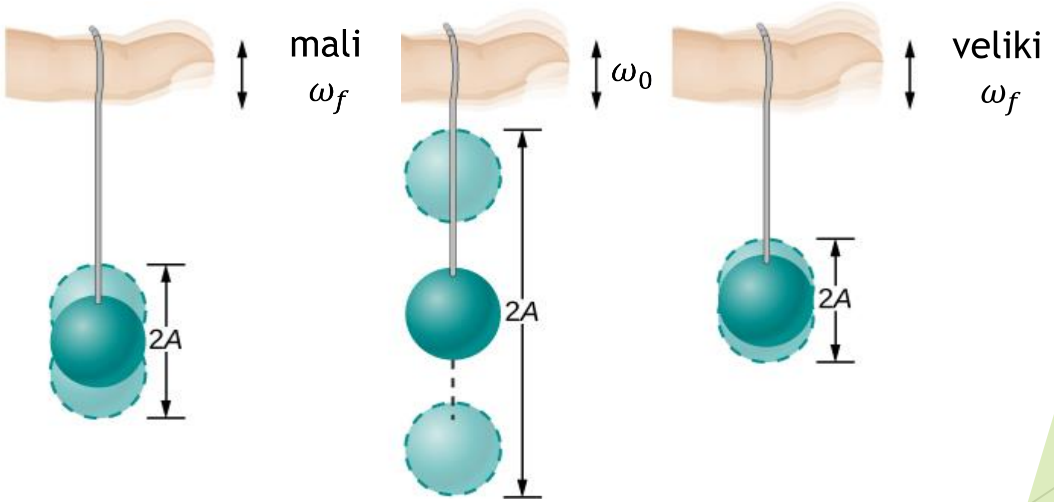
- Amplituda je stalna – neprigušeno titranje, ali ovisi o frekvenciji vanjske sile:

$$A = A(\omega_f) = \frac{\frac{F_0}{m}}{\sqrt{\left(\omega_f{}^2 - \omega_0{}^2\right)^2 + 4\gamma^2\omega_f{}^2}}$$

Prisilne oscilacije

- Rezonancija – izrazito povećanje amplitude na određenoj frekvenciji.



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