

Ravnoteža kapljevina-krutina

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Draft Tube Crystallizer

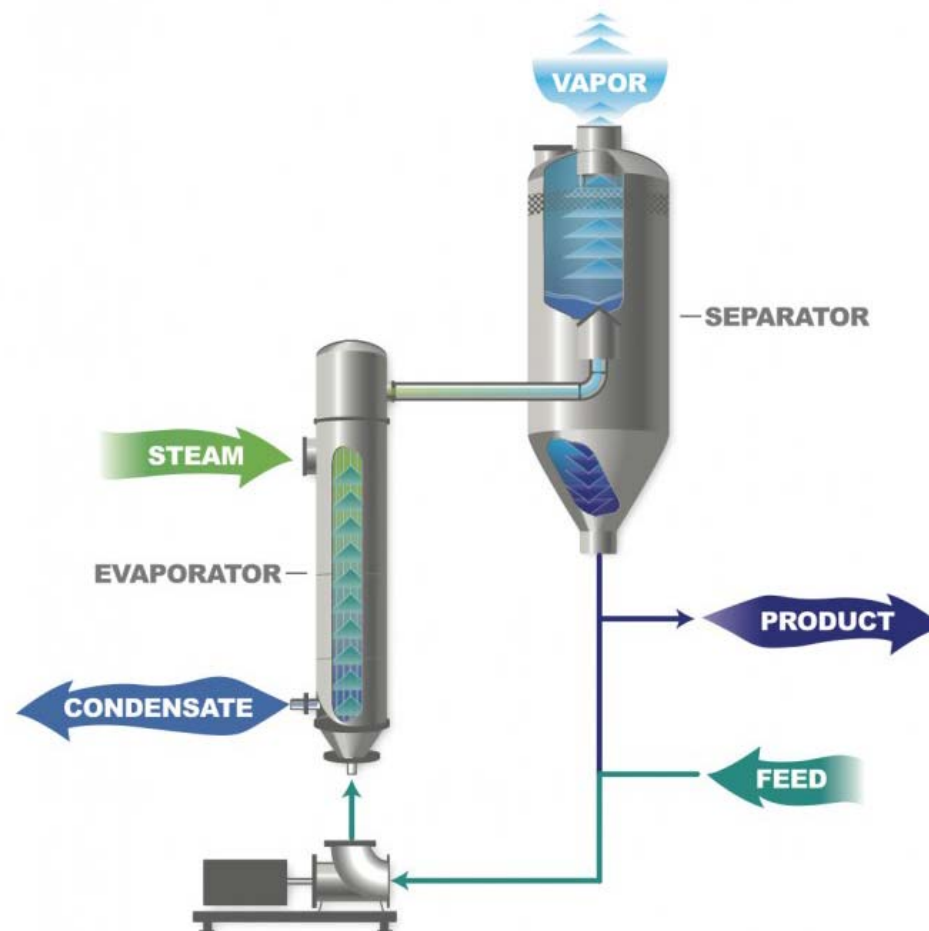
Fazna ravnoteža

Nema kemijskih ili elektrokemijskih procesa

Industrijska praksa

Kristalizacija

Metalurški procesi



Uvjeti fazne ravnoteže

Prvi zakon termodinamike – bilančne jednadžbe

$$\sum_{i=1}^{nk} n_i^F = \sum_{i=1}^{nk} n_i^L + \sum_{i=1}^{nk} n_i^S \quad \text{Ukupna bilanca tvari}$$

$$n^F z_i^F = n^L x_i^L + n^S x_i^S \quad \text{Bilanca po komponentama}$$

$$\sum_{i=1}^{nk} x_i^L = 1 \quad \sum_{i=1}^{nk} x_i^S = 1 \quad \text{Bilanca po fazama}$$

$$H^F = H^S + H^L \quad \text{Bilanca energije za izolirani sustav}$$

Uvjeti fazne ravnoteže

Drugi zakon termodinamike

$$S = \max. \quad dS = 0 \quad \text{Izolirani sustav}$$

Ekvivalentni uvjeti za dvofazne, višekomponentne sustave

$$T^L = T^S$$

$$p^L = p^S$$

$$\mu_i^L = \mu_i^S$$

Jednadžba fazne ravnoteže

$$\mu_i^L = \mu_i^S \quad \text{Jednakost kemijskih potencijala}$$

$$(\mu_i^L - \mu_i^S)_T = RT \ln \frac{\hat{f}_i^L}{\hat{f}_i^S} = 0 \quad \text{Preko parcijalnih fugacitivnosti}$$

$$\hat{f}_i^L = \hat{f}_i^S \quad \text{Jednakost parcijalnih fugacitivnosti}$$

$$a_i = \frac{\hat{f}_i^L}{f_i^{L^\circ}} \quad a_i = \frac{\hat{f}_i^S}{f_i^{S^\circ}} \quad \text{Uvođenje aktivnosti} \quad a_i = \gamma_i x_i$$

Uvođenje koeficijenta aktivnosti

$$x_i^L \gamma_i^L f_i^{L^\circ} = x_i^S \gamma_i^S f_i^{S^\circ} \quad \text{Ravnotežna jednadžba}$$

Jednadžba fazne ravnoteže

NA STRANI KAPLJEVITE FAZE

Za komponentu koja pri temperaturi i tlaku sustava može postojati kao kapljevina (otapalo)

$$f_i^{L\circ} = f_i^\circ \exp \left[\int_{p_i^\circ}^p \frac{v_i^L}{RT} dp \right]$$

$$f_i^\circ = \varphi_i^\circ p_i^\circ \quad \varphi_i^\circ = 1$$

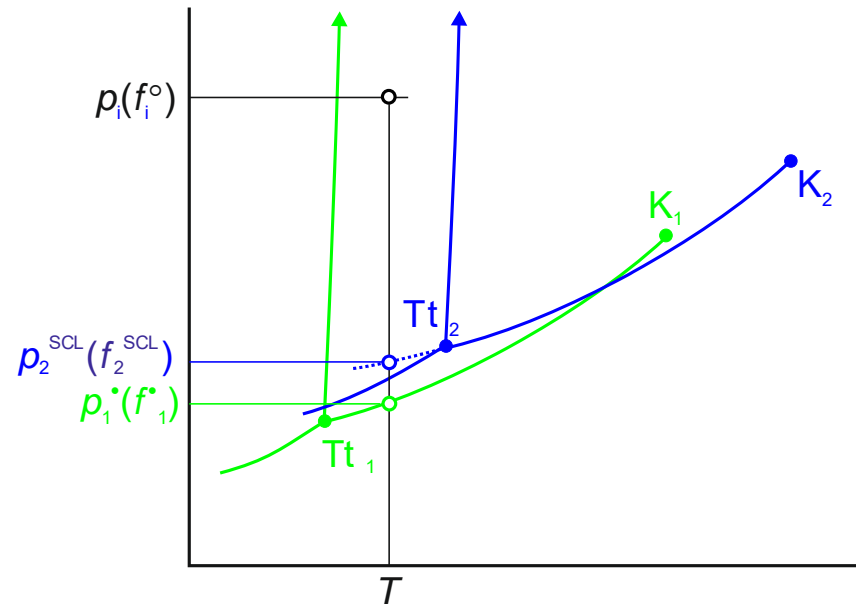
Standardna fugacitivnost (čiste) kapljevine

Potrebno:

Ravnotežni tlak para

Molarni volumen kapljevine

Jednadžba stanja ?!



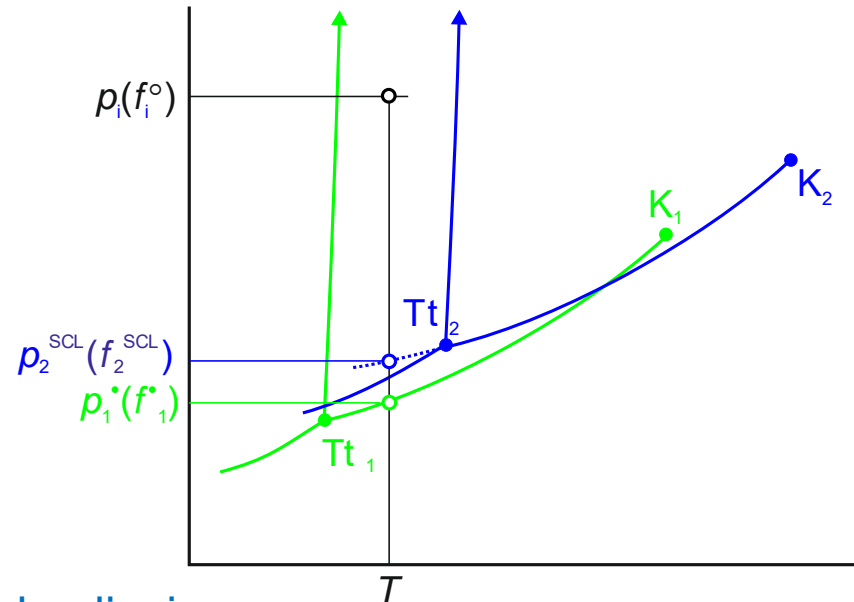
Jednadžba fazne ravnoteže

NA STRANI KAPLJEVITE FAZE

Za komponentu koja pri temperaturi i tlaku sustava **ne može** postojati kao kapljevina (otopljena krutina)

$$f_i^{\text{SCL}\circ} = f_i^\circ \exp \left[\int_{p_i^\circ}^p \frac{v_i^L}{RT} dp \right]$$

$$f_i^\circ = \varphi_i^\circ p_i^\circ \quad \varphi_i^\circ = 1$$



Standardna fugacitivnost (čiste) **pothlađene** kapljevine
(**sub-cooled, SCL**)

Potrebno:

Hipotetski ravnotežni tlak para

Hipotetski molarni volumen kapljevine

Jednadžba stanja ?!

Jednadžba fazne ravnoteže

NA STRANI ČVRSTE FAZE

Za komponentu koja pri temperaturi i tlaku sustava postoji kao krutina (neotopljeni kristali)

$$f_i^{S_0} = f_i^\bullet \exp \left[\int_{p_i^\bullet}^p \frac{v_i^S}{RT} dp \right]$$

$$f_i^\bullet = \varphi_i^\bullet p_i^{S^\bullet} \quad \varphi_i^\bullet = 1$$

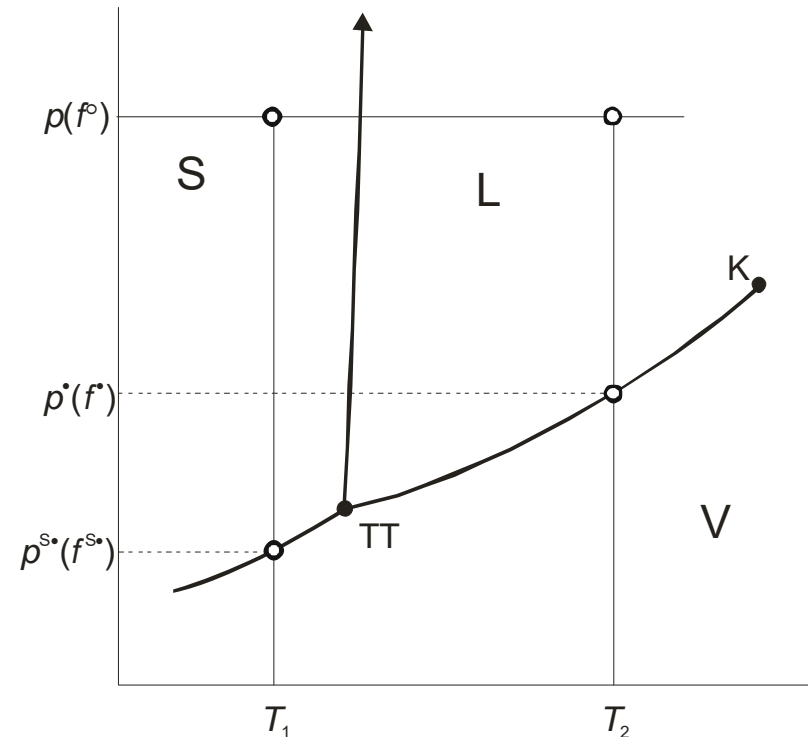
Standardna fugacitivnost (čiste) krutine

Potrebno:

Ravnotežni tlak para sublimacije ?

Molarni volumen krutine

Jednadžba stanja ?!



<https://www.youtube.com/watch?v=jX9pskbKSw0>

<https://www.youtube.com/watch?v=A2qBnlxWhZQ>

<https://answers.yahoo.com/question/index?qid=20090322154450AAFeKHx>

<https://www.youtube.com/watch?v=TEIzO6tpSfQ>

Jednadžba fazne ravnoteže

NA STRANI ČVRSTE FAZE

Za komponentu koja pri temperaturi i tlaku sustava postoji kao krutina (neotopljeni kristali)

Standardna fugacitivnost (čiste) krutine

Potrebno:

Temperatura trojne točke
Tlak trojne točke

Fugacitivnost trojne točke = ?

Molarni volumen krutine

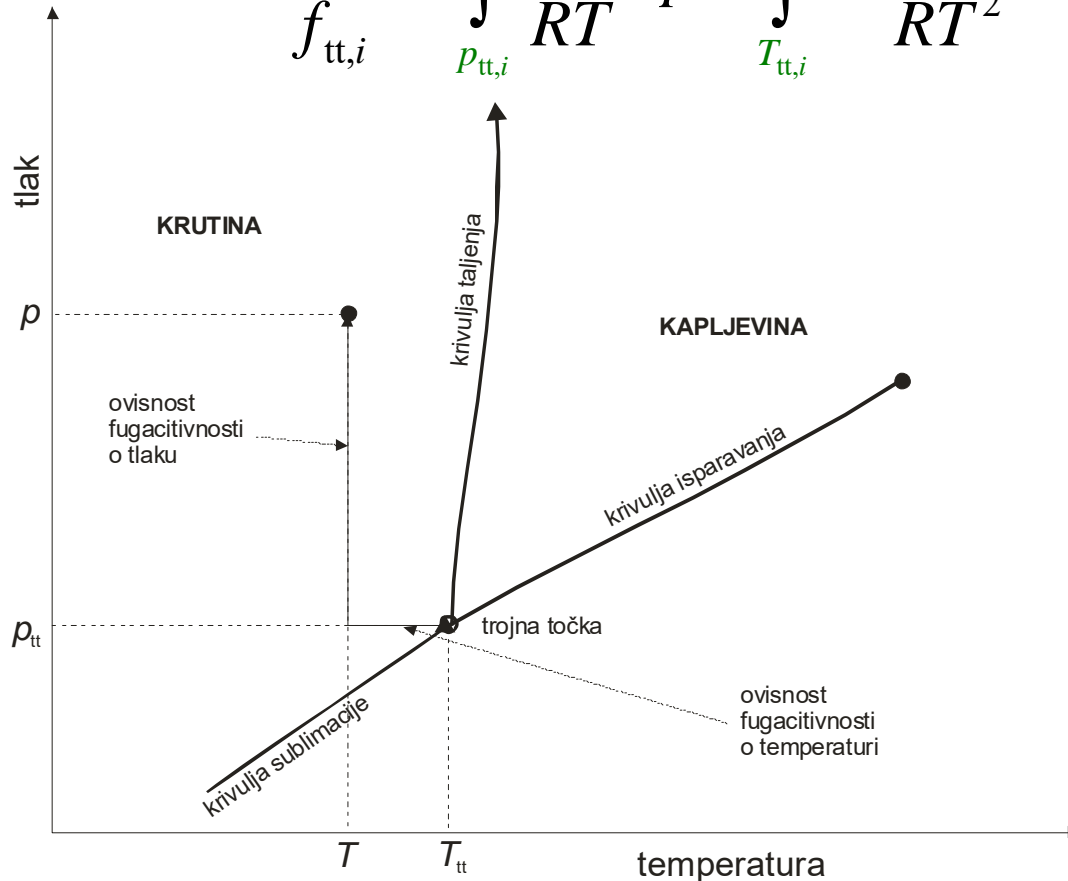
Entalpija sublimacije

$$h_i^{V,id} - h_i^S \approx \Delta h_i^{subl}$$

$$\Delta H_i^{subl} \approx \Delta H_i^{talj} + \Delta H_i^{isp}$$

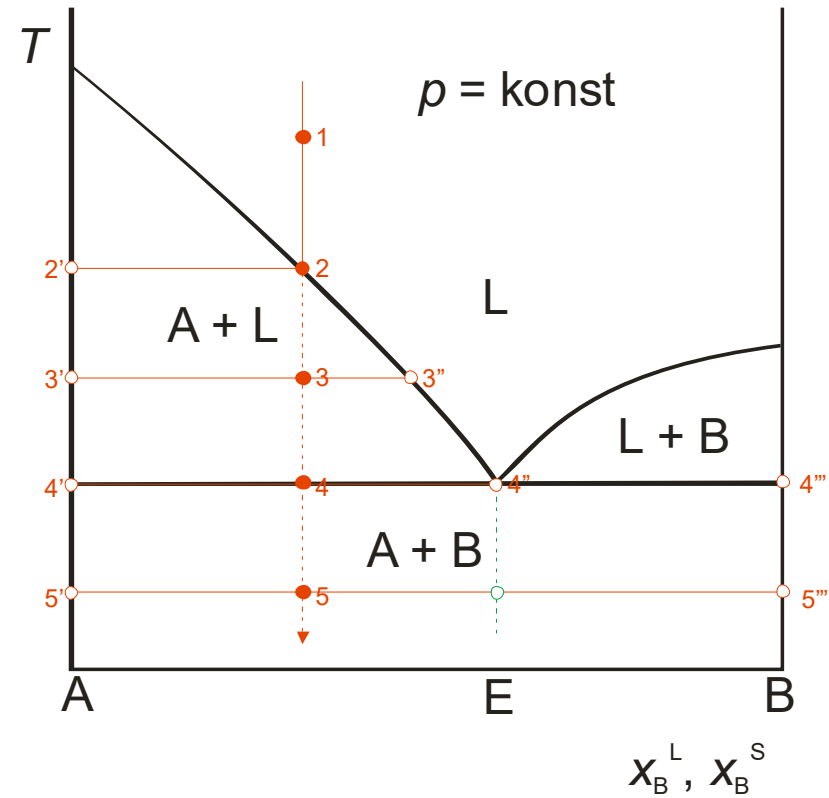
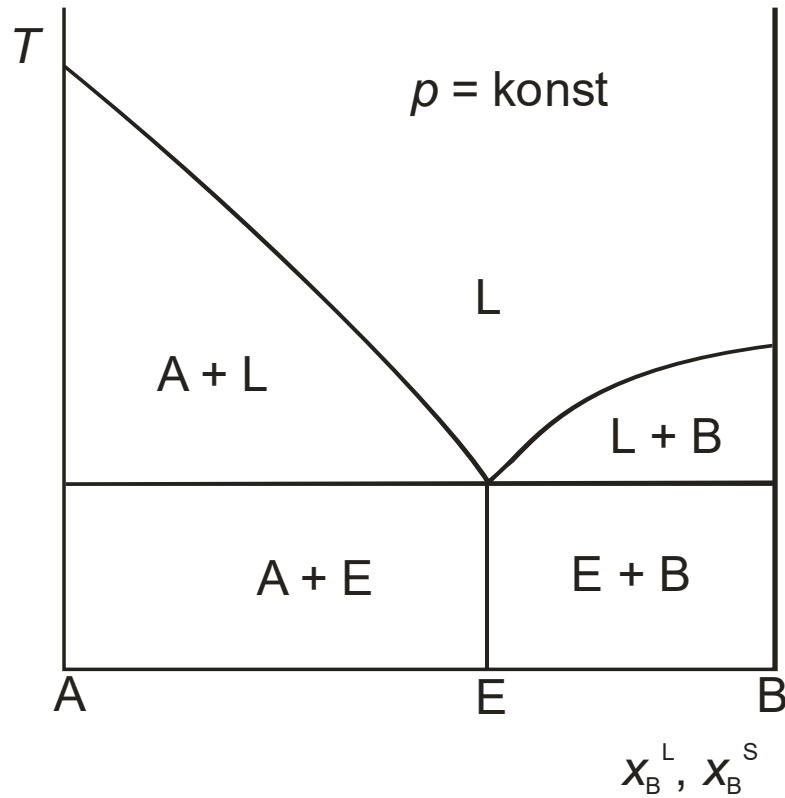
$$f_{tt,i} \approx p_{tt,i}$$

$$\ln \frac{f_i^{S_0}}{f_{tt,i}} = \int_{p_{tt,i}}^p \frac{v_i^S}{RT} dp + \int_{T_{tt,i}}^T \frac{h_i^{V,id} - h_i^S}{RT^2} dT$$



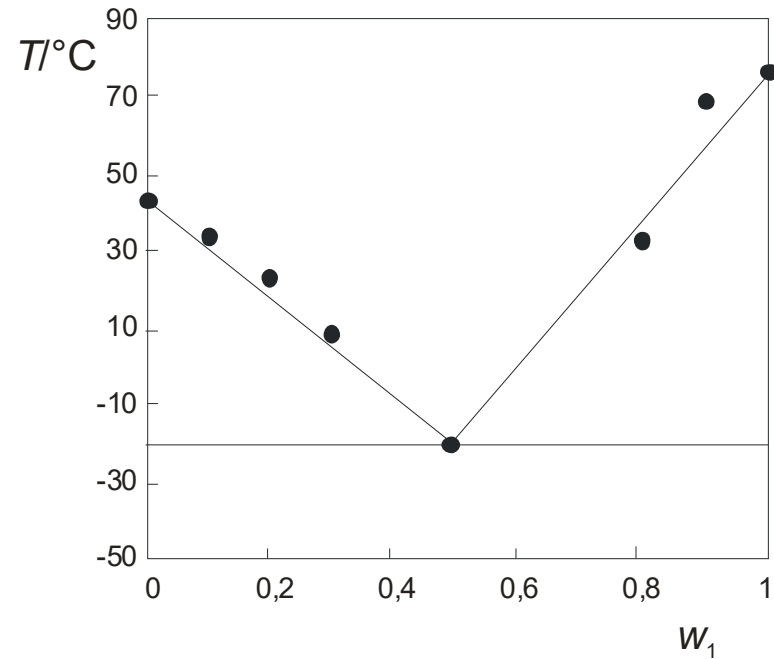
Fazni dijagrami

Potpuna nemješljivost



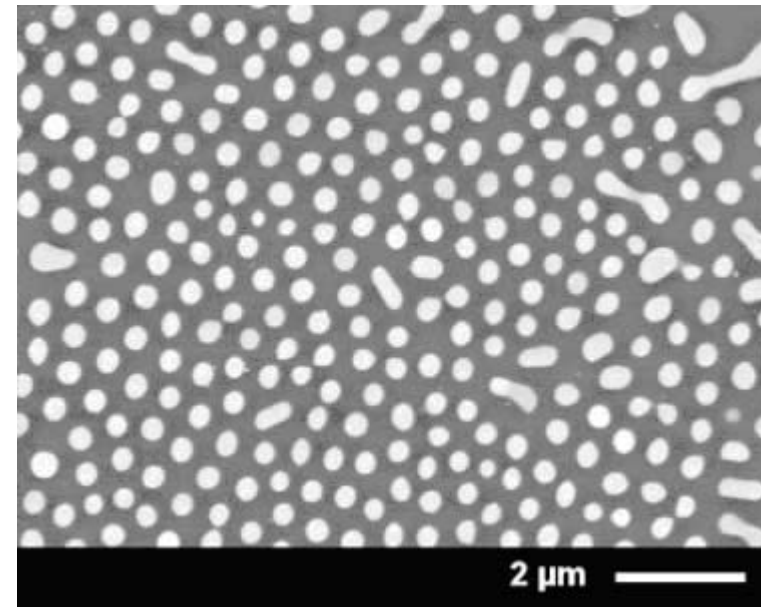
Fazni dijagrami

Potpuna nemješljivost



ibuprofen(1) – metil-nikotinat(2), 10325 Pa, US Patent 6841161

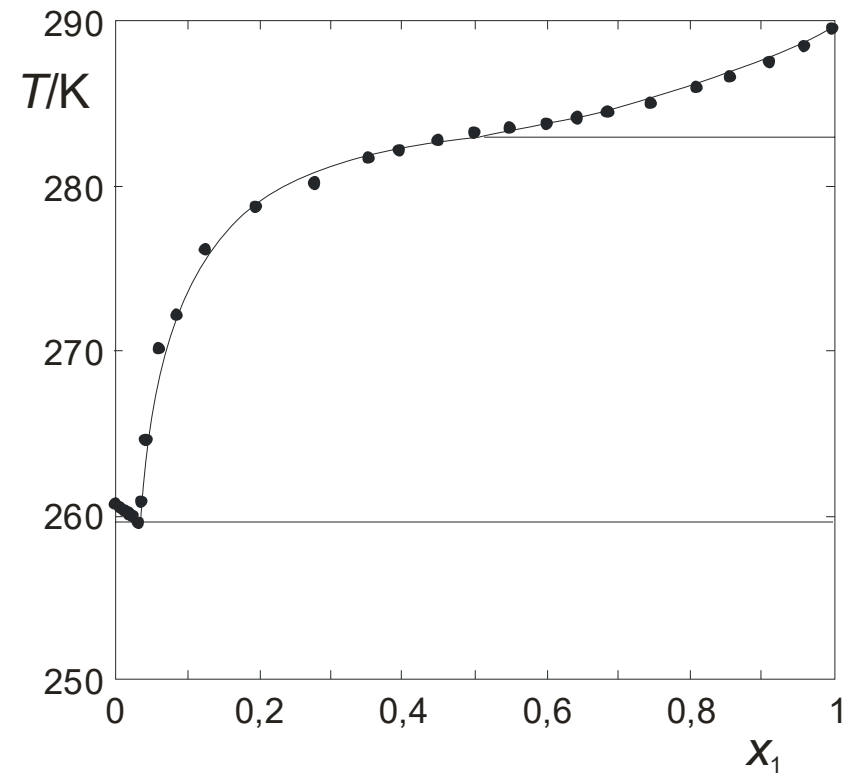
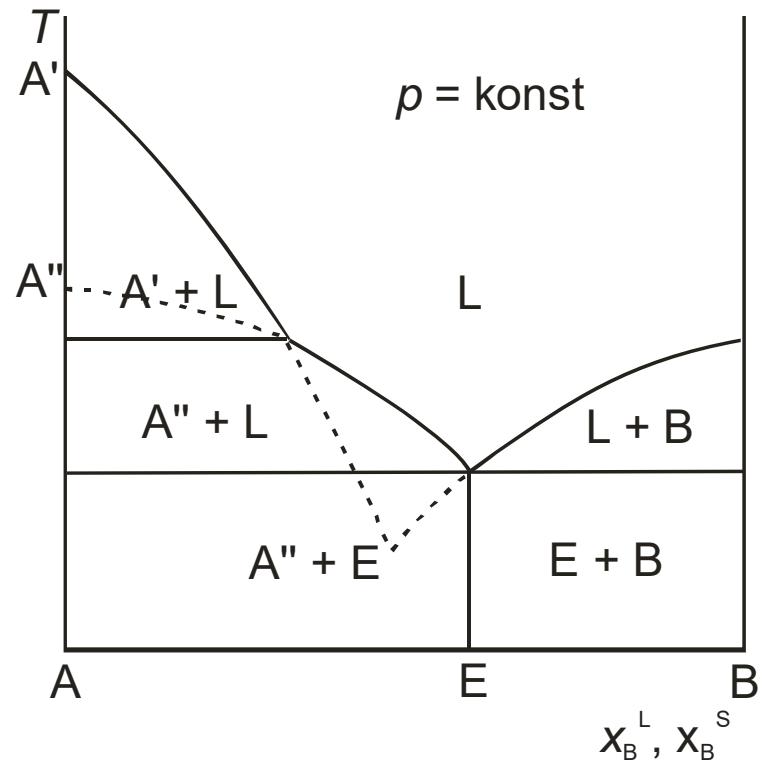
Eutektik



Nb (81.8%) – Si (18.2%) legura
Nb svijetla područja
Nb₃Si tamna područja

Fazni dijagrami

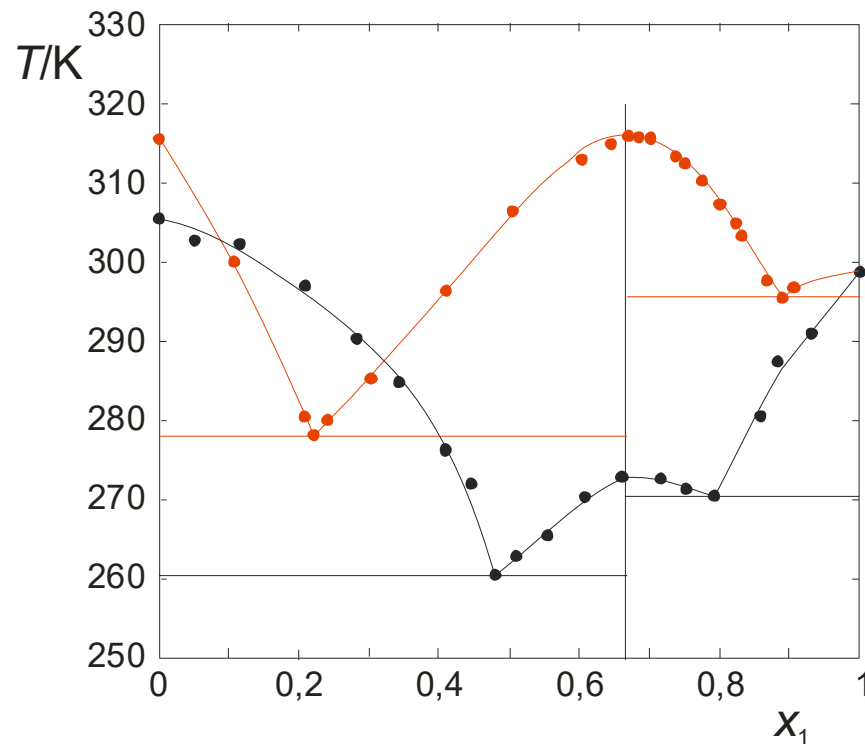
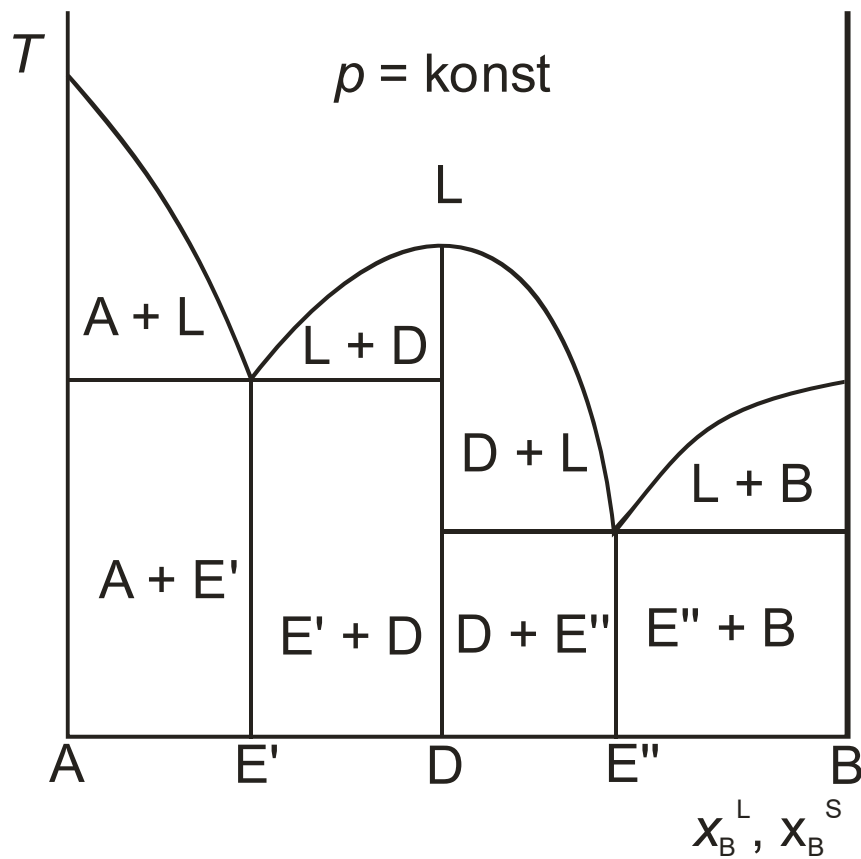
Potpuna nemješljivost, **dvije kristalne modifikacije**



undekan-1-ol(1) i benzonitril(2), U. Domańska i M. Marciniak

Fazni dijagrami

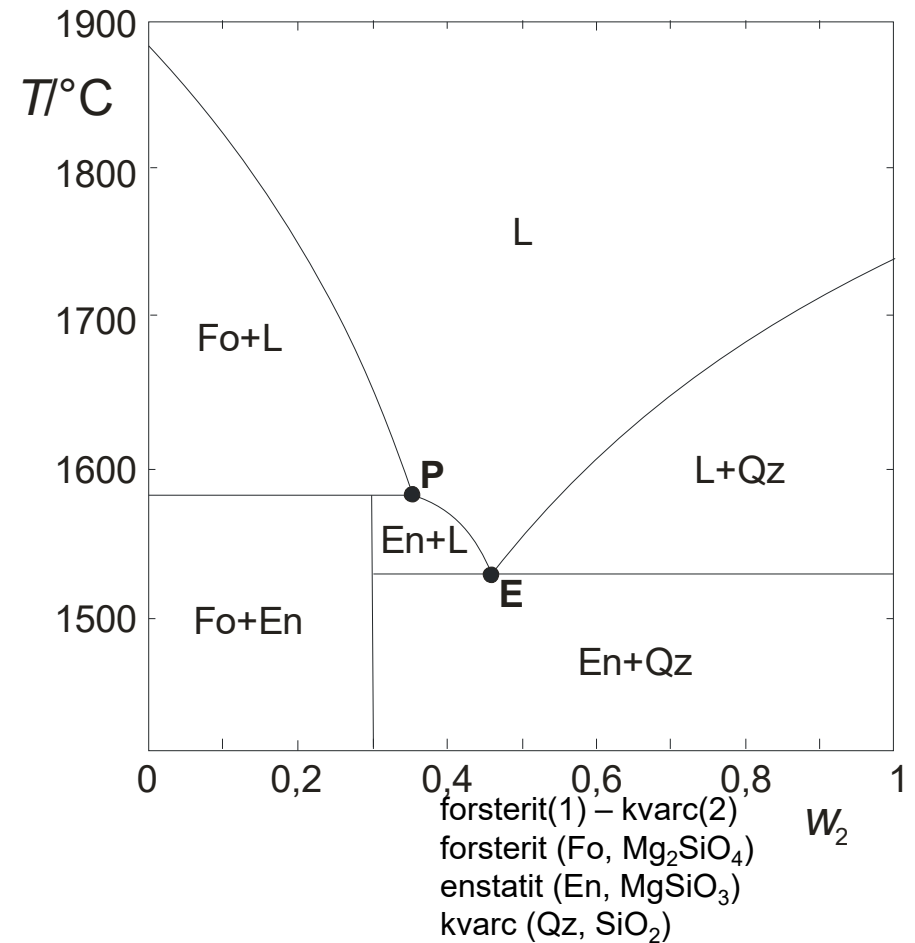
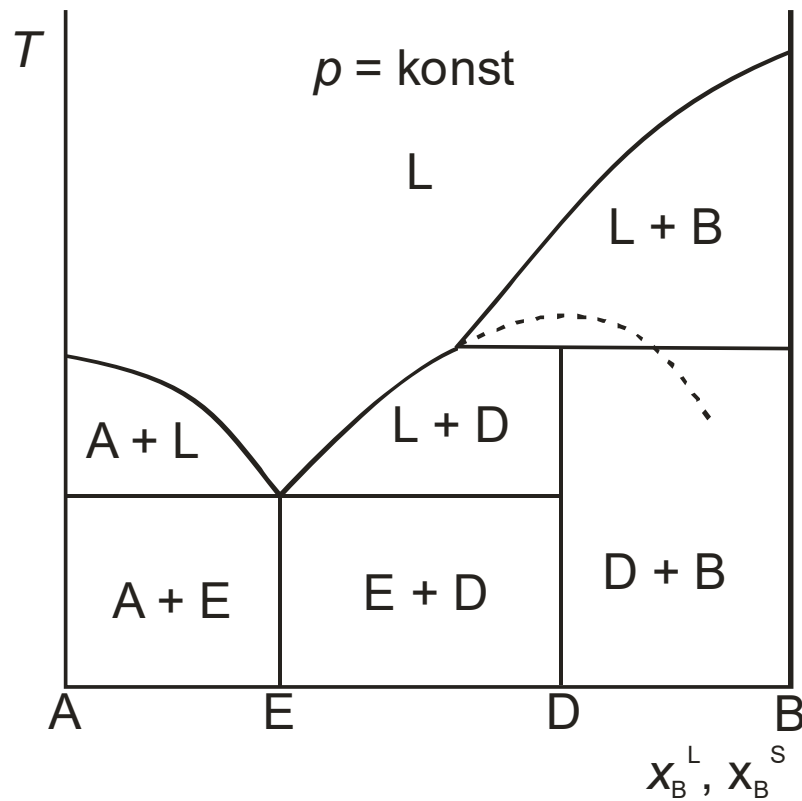
Potpuna nemješljivost, međumolekulski spoj, dva eutektika



terc-butanol(1) – *m*-klorofenol(2)
terc-butanol(1) – *p*-klorofenol(2)
 T.-M. Her i suradnici

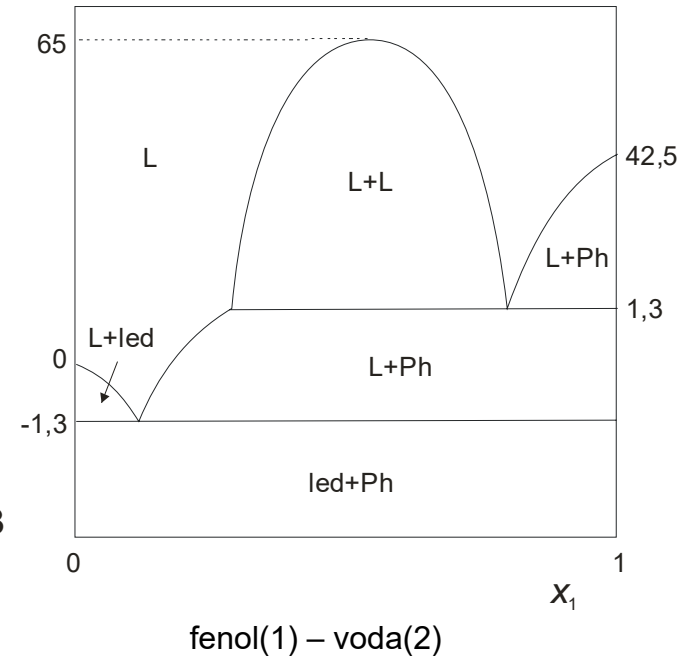
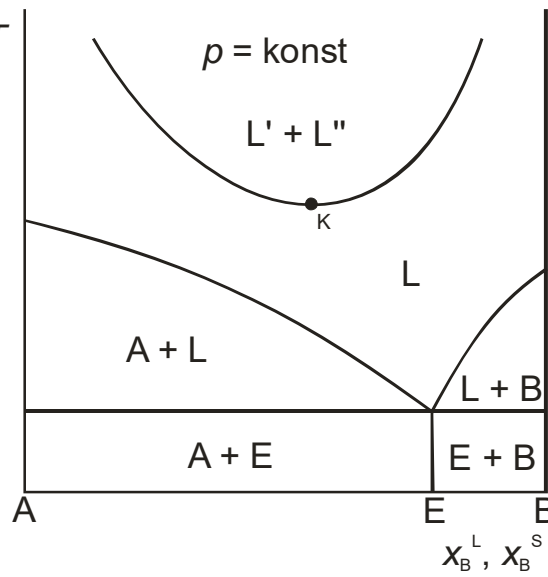
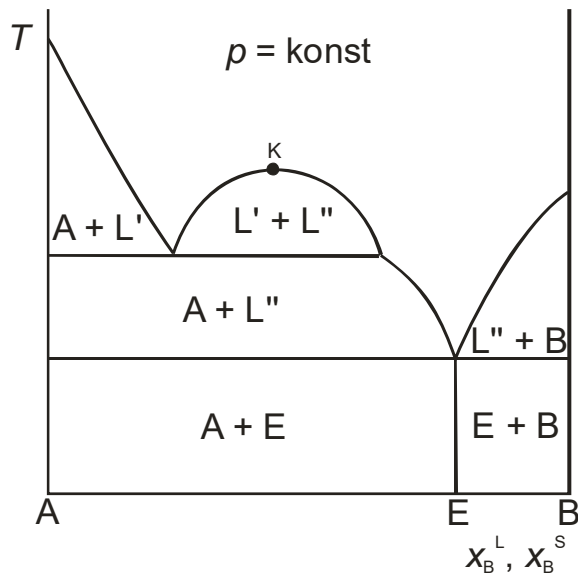
Fazni dijagrami

Potpuna nemješljivost, međumolekulski spoj, eutektik, **peritektik**, **peritektička transformacija (reakcija)**, **nekongruentno taljenje**



Fazni dijagrami

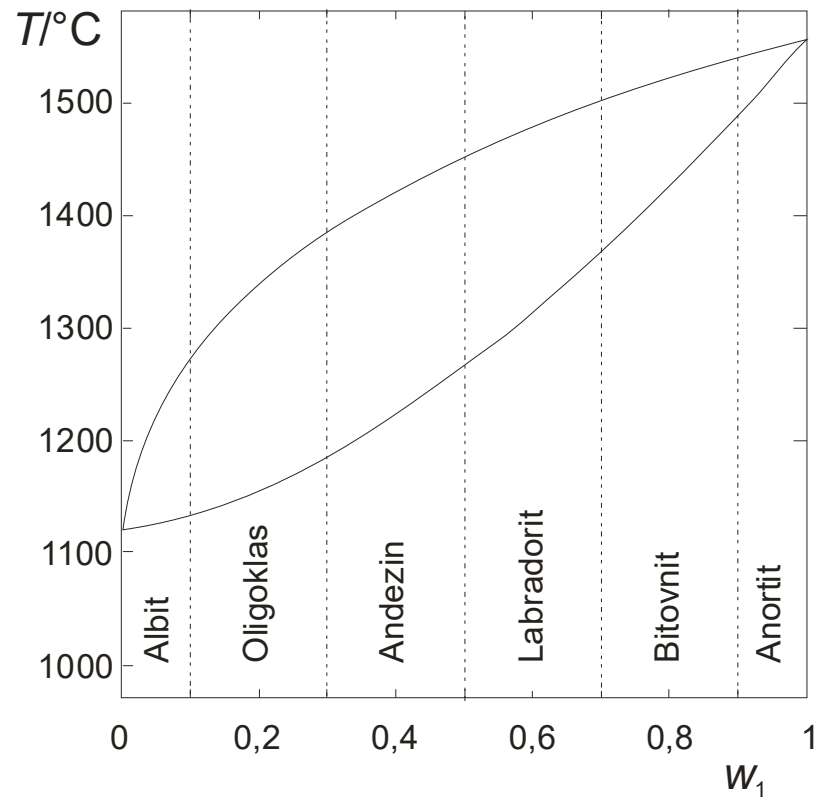
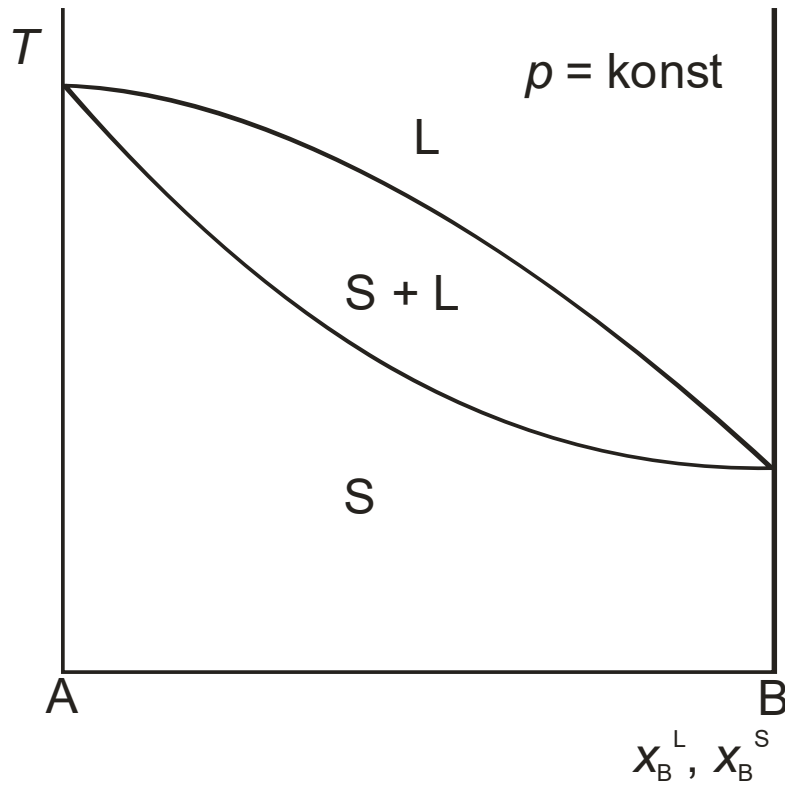
Djelimična mješljivost u kapljevitofazi



fenol(1) – voda(2)

Fazni dijagrami

Potpuna mješljivost u kapljevitj i krutoj fazi

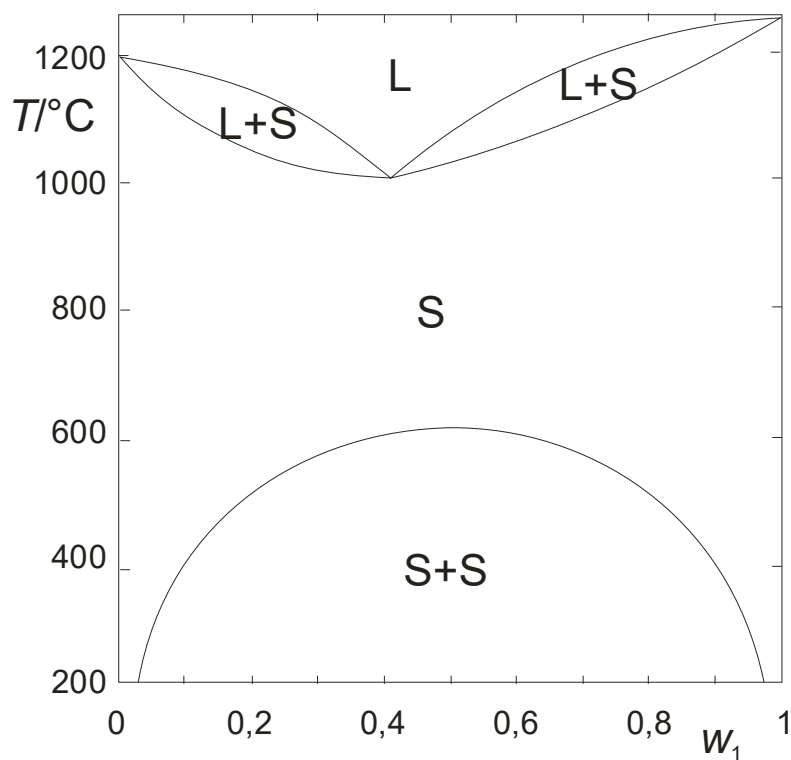


anortit($\text{CaAl}_2\text{Si}_2\text{O}_8$, 1) – albit($\text{NaAlSi}_3\text{O}_8$, 2)

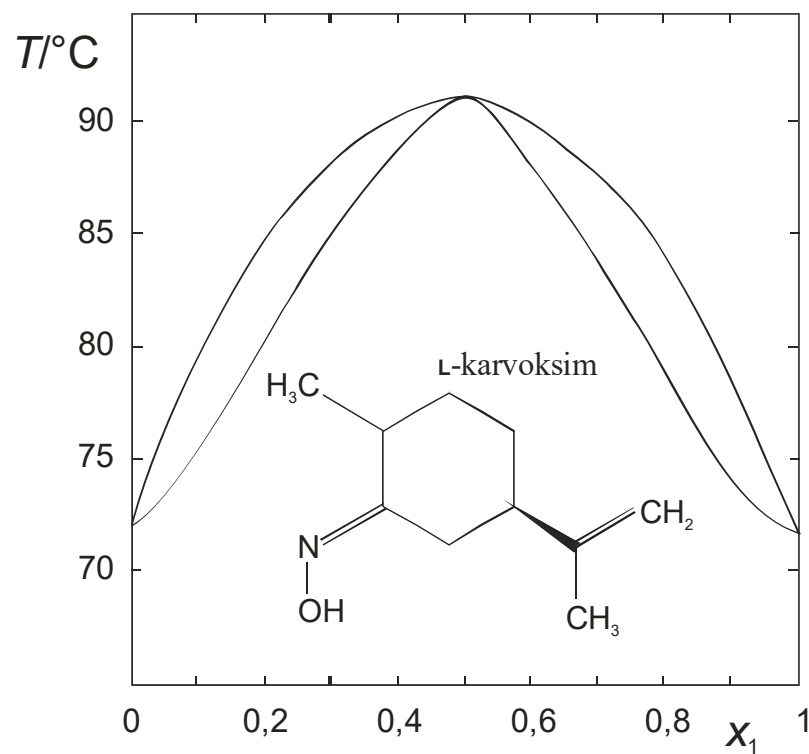
Fazni dijagrami

Potpuna mješljivost u kapljevitj i krutoj fazi

Minimum ili maksimum tališta ("azeotropija")



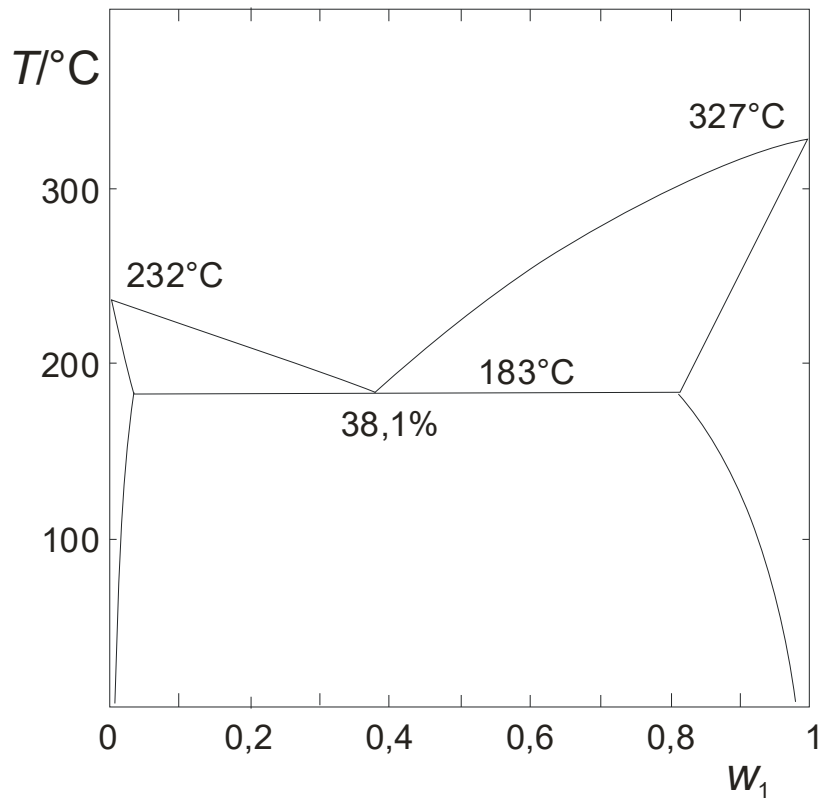
ortoklas(KAlSi₃O₈, 1) – albit(NaAlSi₃O₈, 2)



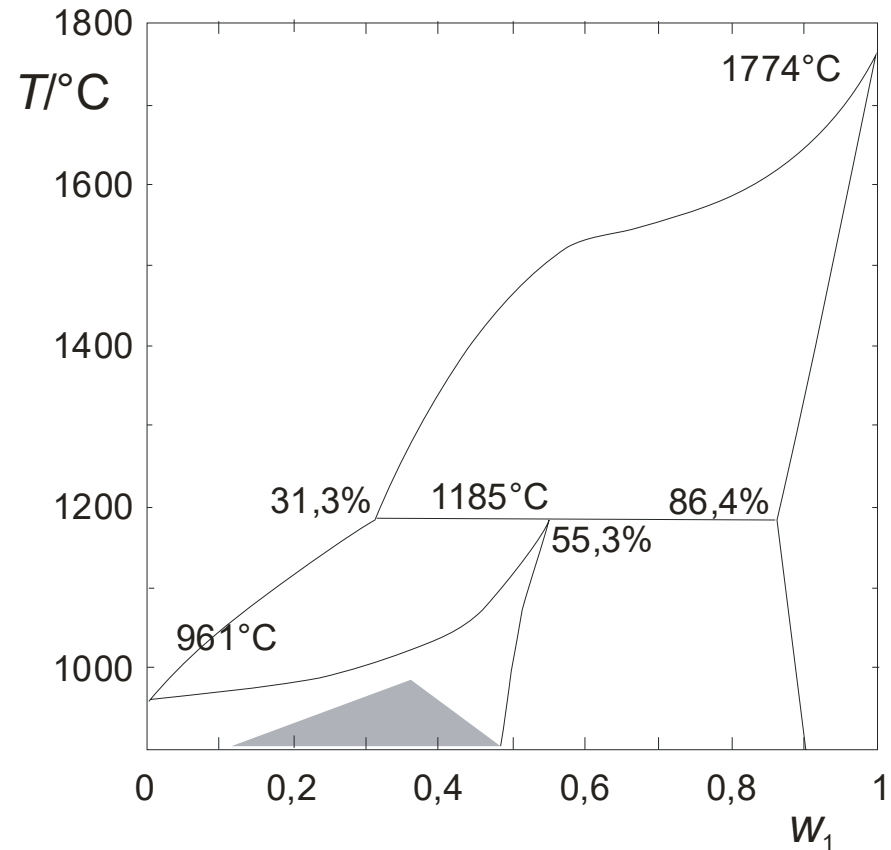
L-karvoksim(1) – D-karvoksim(1)

Fazni dijagrami

Djelimična mješljivost u krutoj fazi



olovo(1) – kositar(2)



platina(1) – srebro(2)

Topljivost krutina

Jednadžba ravnoteže $\hat{f}_i^L = \hat{f}_i^S$

$$\hat{f}_2^L = \hat{f}_2^S$$

Otopljena tvar

Standardno stanje čiste
pothlađene kapljevine

$$\hat{f}_2^L = x_2^L \gamma_2^L f_2^{\text{SCL}\circ} \quad \hat{f}_2^S = f_2^S$$

Krutina je čista tvar

$$x_2^L = \frac{1}{\gamma_2^L} \cdot \frac{f_2^S}{f_2^{\text{SCL}\circ}}$$

Jednadžba topljivosti

$$f_2^{\text{SCL}\circ} = \varphi_2^{\text{SCL}\circ} p_2^{\text{SCL}\circ} \exp \left[\int_{p_2^{\text{SCL}\circ}}^p \frac{v_2^L}{RT} dp \right]$$

Kapjevina

$$f_2^S = \varphi_2^{\text{S}\circ} p_2^{\text{S}\circ} \exp \left[\int_{p_2^{\text{S}\circ}}^p \frac{v_2^S}{RT} dp \right]$$

Krutina

Topljivost krutina

$$f_2^{\text{SCL}\circ} = \varphi_2^{\text{SCL}\bullet} p_2^{\text{SCL}\bullet} \exp\left[\frac{v_2^{\text{L}}(p - p_2^{\text{SCL}\bullet})}{RT}\right]$$

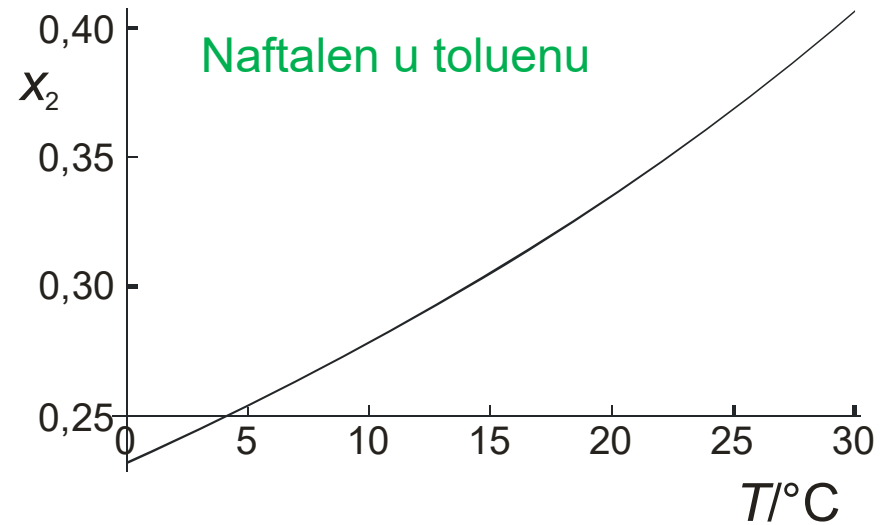
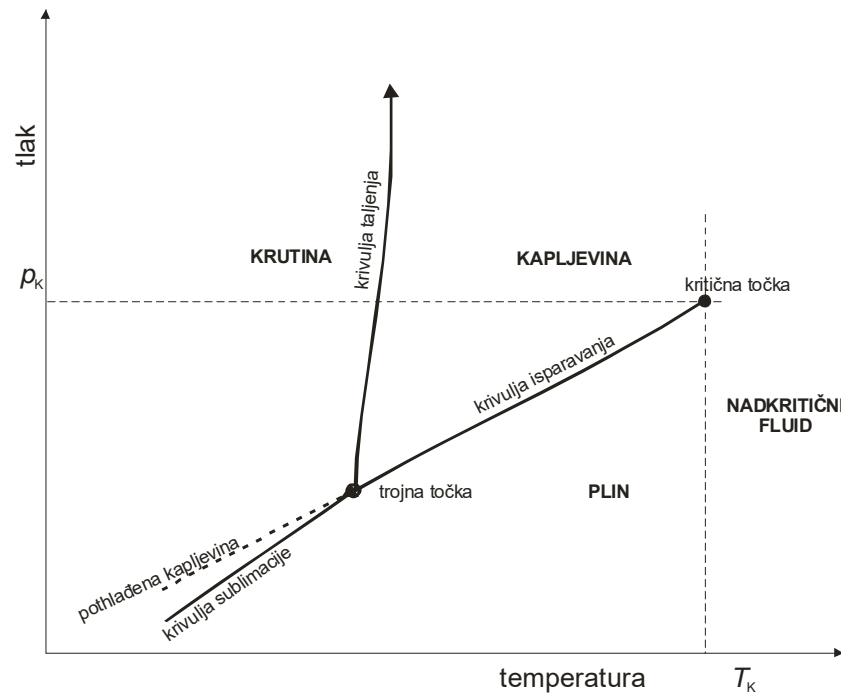
$$f_2^{\text{S}} = \varphi_2^{\text{S}\bullet} p_2^{\text{S}\bullet} \exp\left[\frac{v_2^{\text{S}}(p - p_2^{\text{S}\bullet})}{RT}\right]$$

$PF_2^{\text{SCL}} \rightarrow 1$	Zanemarivanje Poyntingova faktora	$PF_2^{\text{S}} \rightarrow 1$
$\varphi_2^{\text{S}\bullet} \rightarrow 1$	Idealna Parna faza	$\varphi_2^{\text{SCL}\bullet} \rightarrow 1$

$$x_2^{\text{L}} = \frac{1}{\gamma_2^{\text{L}}} \cdot \frac{p_2^{\text{S}\bullet}}{p_2^{\text{SCL}\bullet}}$$

Jednadžba topljivosti

Topljivost krutina



$$\log(p_2^{\text{SCL}} / \text{bar}) = 4,13555 - \frac{1733,710}{T + 201,859 - 273,15}$$

$$\log(p_2^{\text{S}} / \text{bar}) = 8,583 - \frac{3733,9}{T}$$

Topljivost krutina

$$x_2^L = \frac{1}{\gamma_2^L} \cdot \frac{f_2^S}{f_2^{\text{SCL}\circ}}$$

Jednadžba topljivosti

$$d \ln f = \frac{v}{RT} dp - \frac{h - h^\circ}{RT^2} dT$$

$$d \ln f_2^S = \frac{v_2^S}{RT} dp - \frac{h_2^S - h_2^\circ}{RT^2} dT \quad \text{Krutina}$$

$$d \ln f_2^{\text{SCL}\circ} = \frac{v_2^{\text{SCL}\circ}}{RT} dp - \frac{h_2^{\text{SCL}\circ} - h_2^\circ}{RT^2} dT \quad \text{Pothlađena kapljevine}$$

$$d \ln \frac{f_2^S}{f_2^{\text{SCL}\circ}} = \frac{h_2^{\text{SCL}\circ} - h_2^S}{RT^2} dT - \frac{v_2^{\text{SCL}\circ} - v_2^S}{RT} dp$$

Molarna
entalpija
taljenja

Molarni
volumen
taljenja

Topljivost krutina

$$h_2^{\text{SCL}\circ} - h_2^{\text{S}} \approx \Delta h_{2,\text{tt}}^{\text{talj}} + \Delta c_{p2,\text{tt}}^{\text{talj}} (T - T_{\text{tt}})$$

$$d \ln \frac{f_2^{\text{S}}}{f_2^{\text{SCL}\circ}} = \frac{\Delta h_{2,\text{tt}}^{\text{talj}} + \Delta c_{p2,\text{tt}}^{\text{talj}} (T - T_{\text{tt}})}{RT^2} dT - \frac{\Delta v_2^{\text{talj}}}{RT} dp$$

Nakon integriranja:

$$\ln \frac{f_2^{\text{S}}}{f_2^{\text{SCL}\circ}} = \frac{\Delta h_{2,\text{tt}}^{\text{talj}}}{R} \left(\frac{1}{T_{\text{tt}}} - \frac{1}{T} \right) - \frac{\Delta c_{p2,\text{tt}}^{\text{talj}}}{R} \left(\ln \frac{T_{\text{tt}}}{T} - \frac{T_{\text{tt}}}{T} + 1 \right) - \frac{\Delta v_2^{\text{talj}}}{RT} (p - p_{\text{tt}})$$

$$x_2^{\text{L}} = \frac{1}{\gamma_2^{\text{L}}} \cdot \frac{\Delta h_{2,\text{tt}}^{\text{talj}}}{R} \left(\frac{1}{T_{\text{tt}}} - \frac{1}{T} \right)$$

$$- \frac{\Delta c_{p2,\text{tt}}^{\text{talj}}}{R} \left(\ln \frac{T_{\text{tt}}}{T} - \frac{T_{\text{tt}}}{T} + 1 \right)$$

$$- \frac{\Delta v_2^{\text{talj}}}{RT} (p - p_{\text{tt}})$$

$$\frac{\Delta v_2^{\text{talj}}}{RT} (p - p_{\text{tt}}) \approx 0$$

$$\Delta c_{p2,\text{tt}}^{\text{talj}} \approx 0$$

Potrebno poznavati:

- model koeficijenta aktivnosti
- trojnu točku (tlak i temperaturu)
- entalpiju taljenja (Δ) u trojnoj točki
- toplinski kapacitet taljenja (Δ) u trojnoj točki
- volumen taljenja (Δ) u trojnoj točki

Topljivost krutina

Potrebno poznavati:

- model koeficijenta aktivnosti
- trojnu točku (tlak i temperaturu)
- entalpiju taljenja (Δ) u trojnoj točki

$$x_2 = \frac{1}{\gamma_2} \exp \left[\frac{\Delta h_{2,tt}^{\text{talj}}}{R} \left(\frac{1}{T_{tt}} - \frac{1}{T} \right) \right]$$

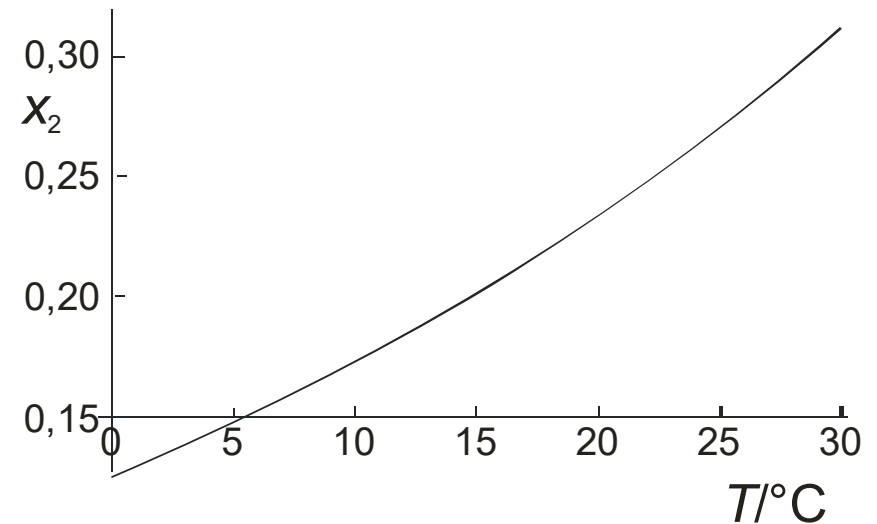
Potrebno poznavati:

- talište pri atmosferskom tlaku
- entalpiju taljenja (Δ) pri atm. tlaku

$$x_2 = \exp \left[\frac{\Delta h_2^{\text{talj}}}{R} \left(\frac{1}{T^{\text{talj}}} - \frac{1}{T} \right) \right]$$

Schröderova jednačba (1893)
Za idealne otopine

Naftalen u toluenu
Model Scatchard-Hildebrand



$$x_2 = \exp \left[\frac{\Delta h_{2,tt}^{\text{talj}}}{R} \left(\frac{1}{T_{tt}} - \frac{1}{T} \right) - \frac{v_2^L}{RT} (\delta_1 - \delta_2)^2 \left(1 + \frac{v_2^L}{v_1^L} \cdot \frac{x_2}{1-x_2} \right)^2 \right]$$

Ravnoteža taljenja

$$x_i^L \gamma_i^L f_i^{L\circ} = x_i^S \gamma_i^S f_i^{S\circ}$$

Ravnotežna enačba
za obje komponente

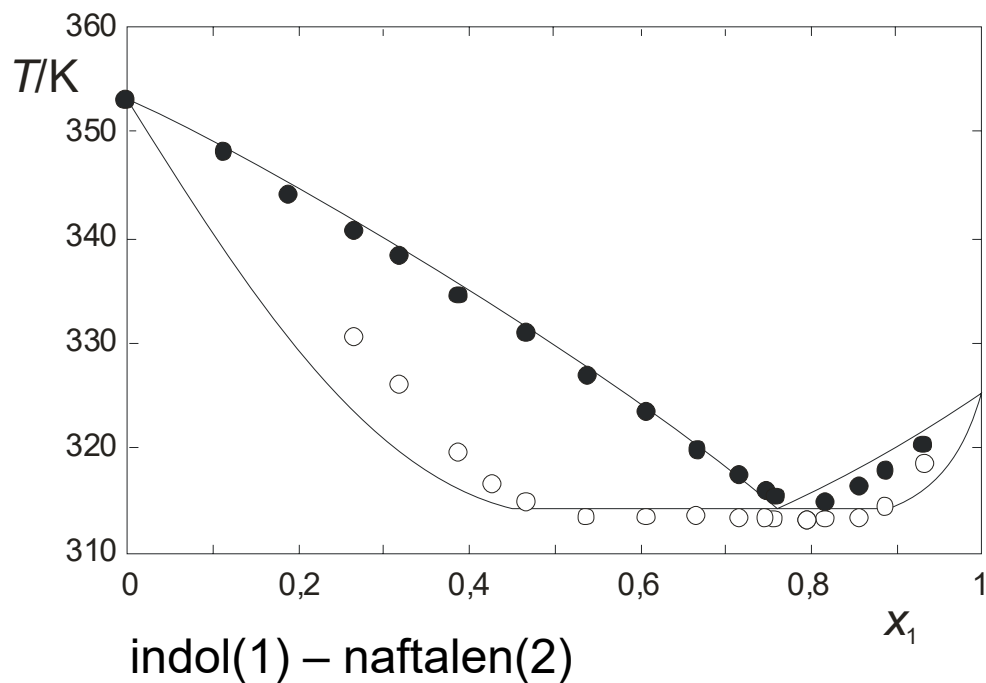
$$\ln \frac{x_i^L \gamma_i^L}{x_i^S \gamma_i^S} = \ln \frac{f_i^{S\circ}}{f_i^{SCL\circ}}$$

$$\ln \frac{f_i^{S\circ}}{f_i^{SCL\circ}} = \frac{\Delta h_{i,tt}^{\text{talj}}}{R} \left(\frac{1}{T_{i,tt}} - \frac{1}{T} \right) - \frac{\Delta c_{pi,tt}^{\text{talj}}}{R} \left(\ln \frac{T_{i,tt}}{T} - \frac{T_{i,tt}}{T} + 1 \right) - \frac{\Delta v_i^{\text{talj}}}{RT} (p - p_{i,tt})$$

$$\ln \frac{\gamma_i^L x_i^L}{\gamma_i^S x_i^S} = \frac{\Delta h_i^{\text{talj}}}{R} \left(\frac{1}{T_i^{\text{talj}}} - \frac{1}{T} \right)$$

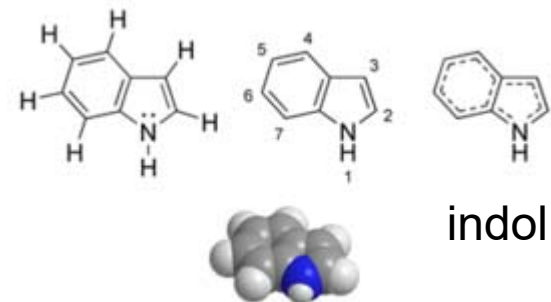
Jednačba slična Schröderovoj
Vrijedi za neidealne otopine

Ravnoteža taljenja



Landolt-Börnstein, Vol. 6, Aufl., Bd.II/2c, Springer, Berlin, 1964.

Model Margules



$$\ln \frac{\gamma_1^L x_1^L}{\gamma_1^S x_1^S} = \frac{\Delta h_1^{\text{talj}}}{R} \left(\frac{1}{T_1^{\text{talj}}} - \frac{1}{T} \right)$$

$$\ln \frac{\gamma_2^L x_2^L}{\gamma_2^S x_2^S} = \frac{\Delta h_2^{\text{talj}}}{R} \left(\frac{1}{T_2^{\text{talj}}} - \frac{1}{T} \right)$$

$$x_1^L + x_2^L = 1$$

$$x_1^S + x_2^S = 1$$

Eutektički uvjeti

$$\ln \frac{\gamma_1^L x_1^L}{\gamma_1^S x_1^S} = \frac{\Delta h_1^{\text{talj}}}{R} \left(\frac{1}{T_1^{\text{talj}}} - \frac{1}{T} \right)$$

$$\ln \frac{\gamma_2^L x_2^L}{\gamma_2^S x_2^S} = \frac{\Delta h_2^{\text{talj}}}{R} \left(\frac{1}{T_2^{\text{talj}}} - \frac{1}{T} \right)$$

$$\ln (\gamma_1^L x_1^L) = \frac{\Delta h_1^{\text{talj}}}{R} \left(\frac{1}{T_1^{\text{talj}}} - \frac{1}{T} \right)$$

$$\ln (\gamma_2^L x_2^L) = \frac{\Delta h_2^{\text{talj}}}{R} \left(\frac{1}{T_2^{\text{talj}}} - \frac{1}{T} \right)$$

$$\ln x_1^L = \frac{\Delta h_1^{\text{talj}}}{R} \left(\frac{1}{T_1^{\text{talj}}} - \frac{1}{T} \right)$$

$$\ln x_2^L = \frac{\Delta h_2^{\text{talj}}}{R} \left(\frac{1}{T_2^{\text{talj}}} - \frac{1}{T} \right)$$

$$x_1^L = \exp \left[\frac{\Delta h_1^{\text{talj}}}{R} \left(\frac{1}{T_1^{\text{talj}}} - \frac{1}{T} \right) \right]$$

$$x_2^L = \exp \left[\frac{\Delta h_2^{\text{talj}}}{R} \left(\frac{1}{T_2^{\text{talj}}} - \frac{1}{T} \right) \right]$$

Schröderove
jednadžbe

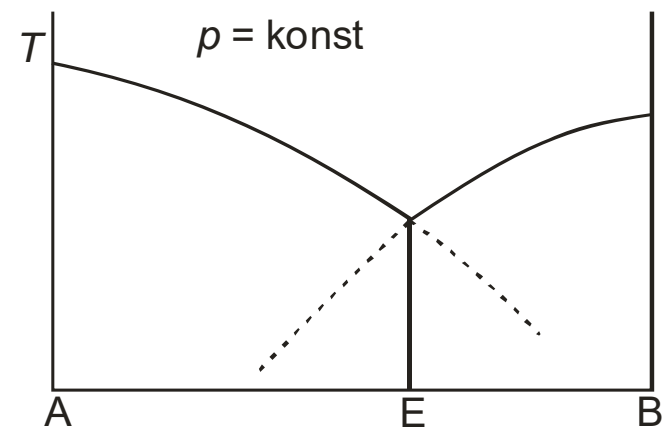
$$T = \left(\frac{1}{T_1^{\text{talj}}} - \frac{R \ln x_1^L}{\Delta h_1^{\text{talj}}} \right)^{-1}$$

$$T = \left(\frac{1}{T_2^{\text{talj}}} - \frac{R \ln x_2^L}{\Delta h_2^{\text{talj}}} \right)^{-1}$$

Jednadžbe tališta
u dvokomponentnom
sustavu

Potpuna
nemješljivost
u krutini

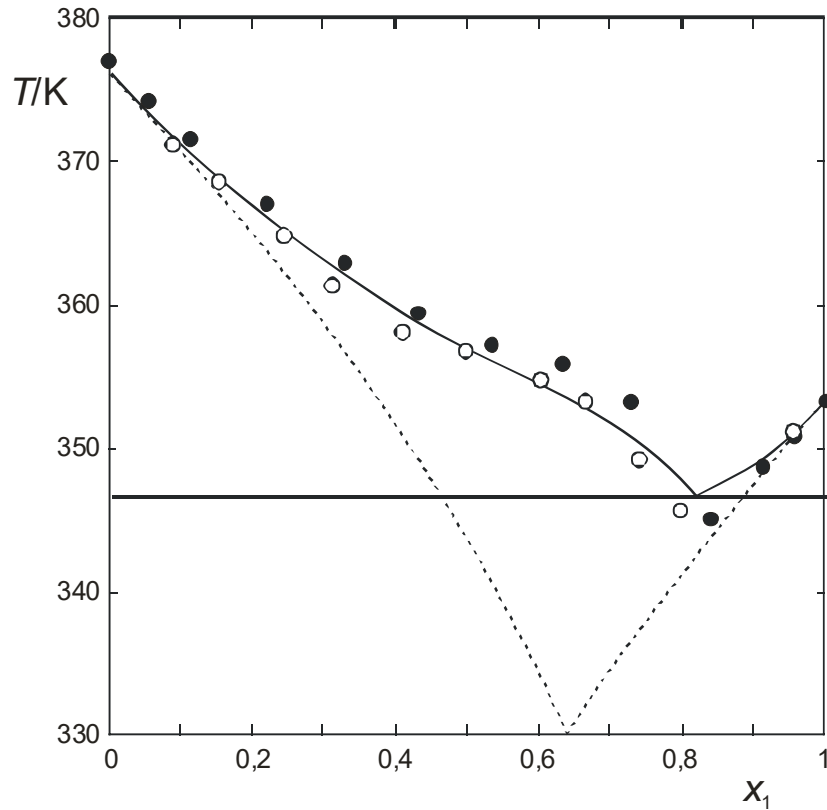
Idealne
otopine



Dijagram s eutektičkom točkom

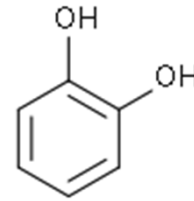
x_B^L, x_B^S

Eutektički uvjeti



naftalen(1) – katehol(2)

Landolt-Börnstein, Vol. 6, Aufl., Bd.II/2c, Springer, Berlin, 1964.



katehol

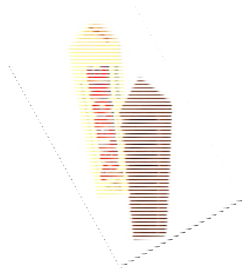
Jednadžbe tališta
u dvokomponentnom
sustavu

$$\ln(\gamma_1^L x_1^L) = \frac{\Delta h_1^{\text{talj}}}{R} \left(\frac{1}{T_1^{\text{talj}}} - \frac{1}{T} \right)$$

$$\ln(\gamma_2^L x_2^L) = \frac{\Delta h_2^{\text{talj}}}{R} \left(\frac{1}{T_2^{\text{talj}}} - \frac{1}{T} \right)$$

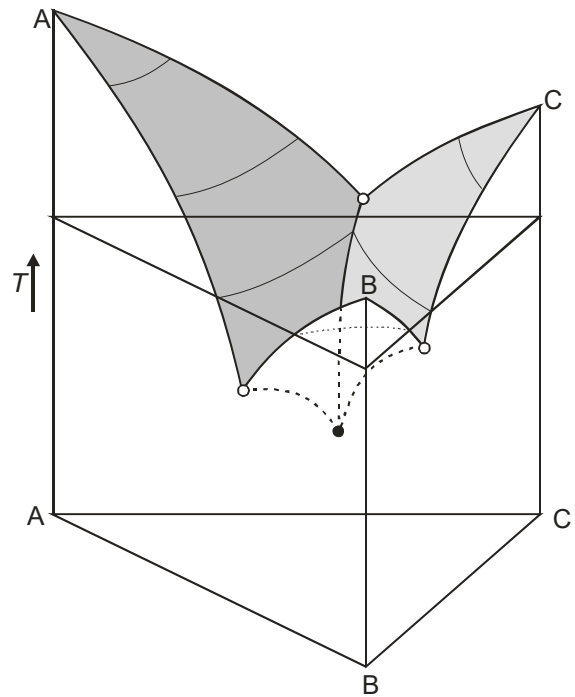
$$x_1^L + x_2^L = 1$$

Model NRTL

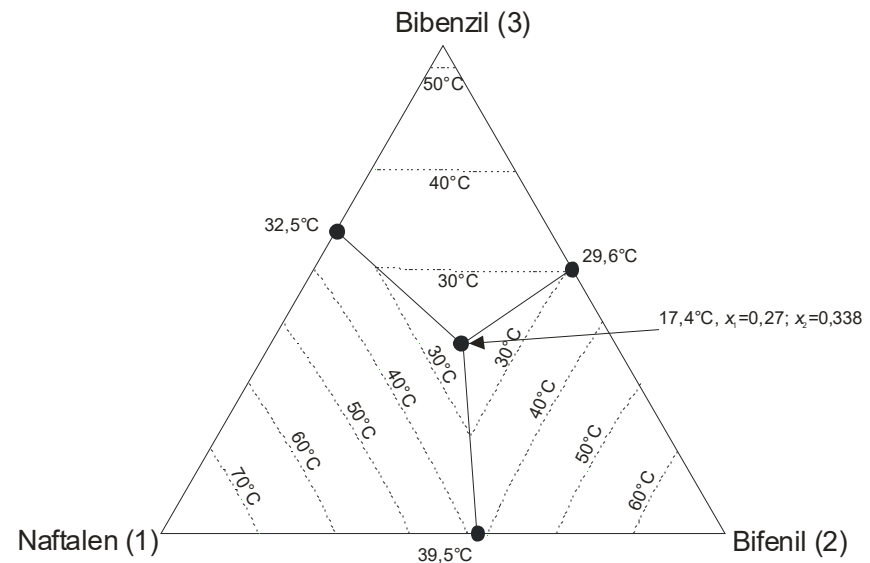
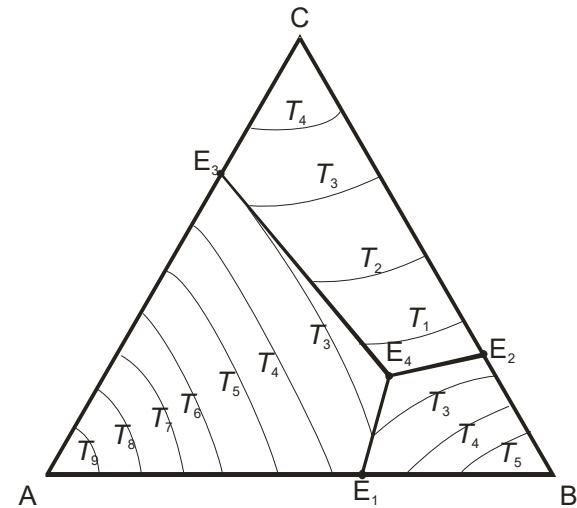


Eutektički uvjeti

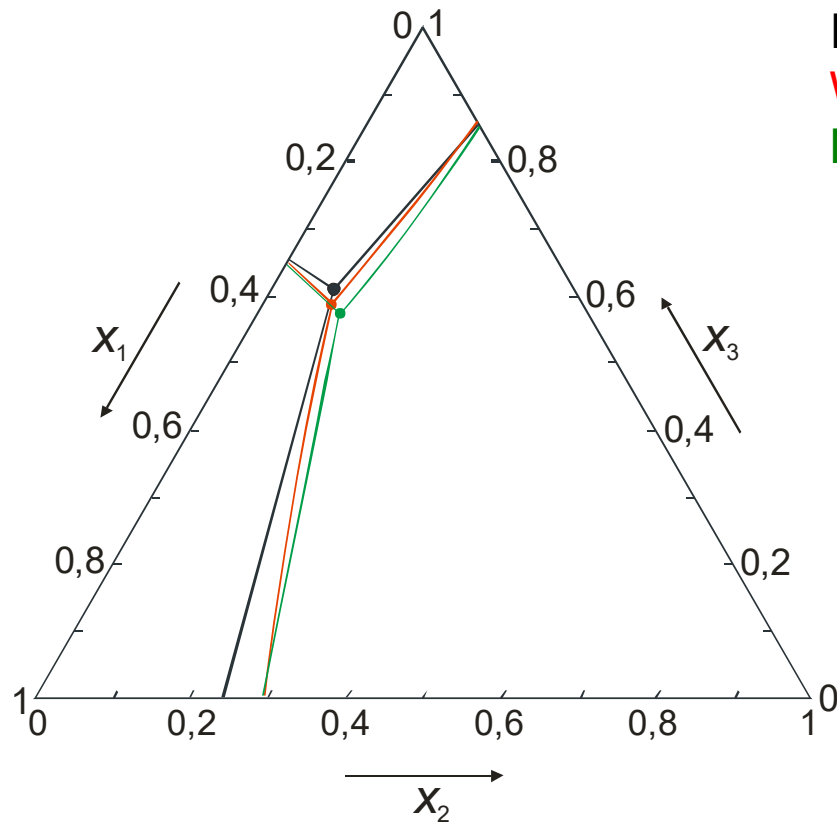
Ternarni eutektik



Eutektički kanal



Eutektički uvjeti



Idealna otopina

Wilson

NRTL

$$x_1 = \frac{1}{\gamma_1^L(x_1, x_2)} \exp \left[\frac{\Delta h_1^{\text{talj}}}{R} \left(\frac{1}{T_1^{\text{talj}}} - \frac{1}{T^e} \right) \right]$$

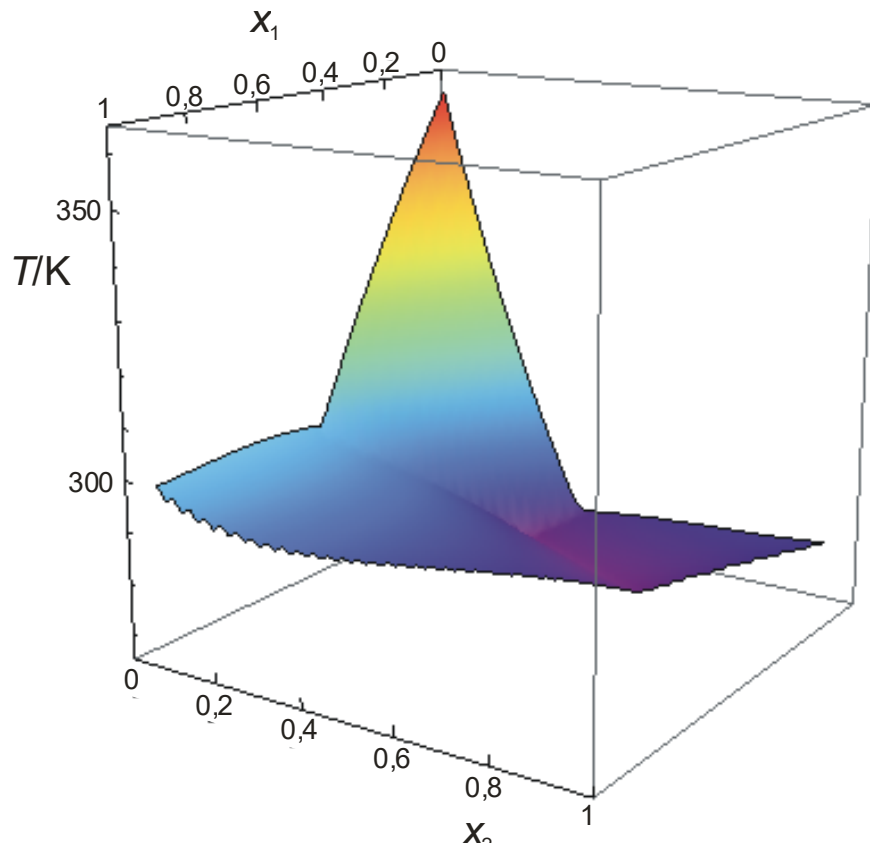
$$x_2 = \frac{1}{\gamma_2^L(x_1, x_2)} \exp \left[\frac{\Delta h_2^{\text{talj}}}{R} \left(\frac{1}{T_2^{\text{talj}}} - \frac{1}{T^e} \right) \right]$$

$$1 - x_1 - x_2 = \frac{1}{\gamma_3^L(x_1, x_2)} \exp \left[\frac{\Delta h_3^{\text{talj}}}{R} \left(\frac{1}{T_3^{\text{talj}}} - \frac{1}{T^e} \right) \right]$$

p-metoksifenol(1) – katehol(2) – *p*-krezol(3)

Ho-mu Lin, Yu-Hsing Chou, Fu-Li Wu, Ming-Jer Lee

Eutektički uvjeti

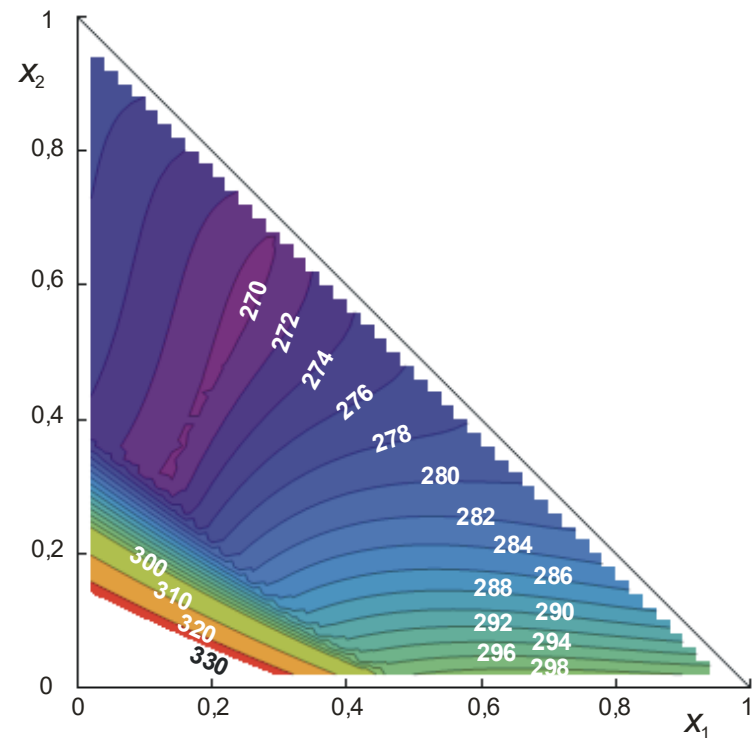


n-dekanol(1) – decilamin(2) – benzonitril(3)

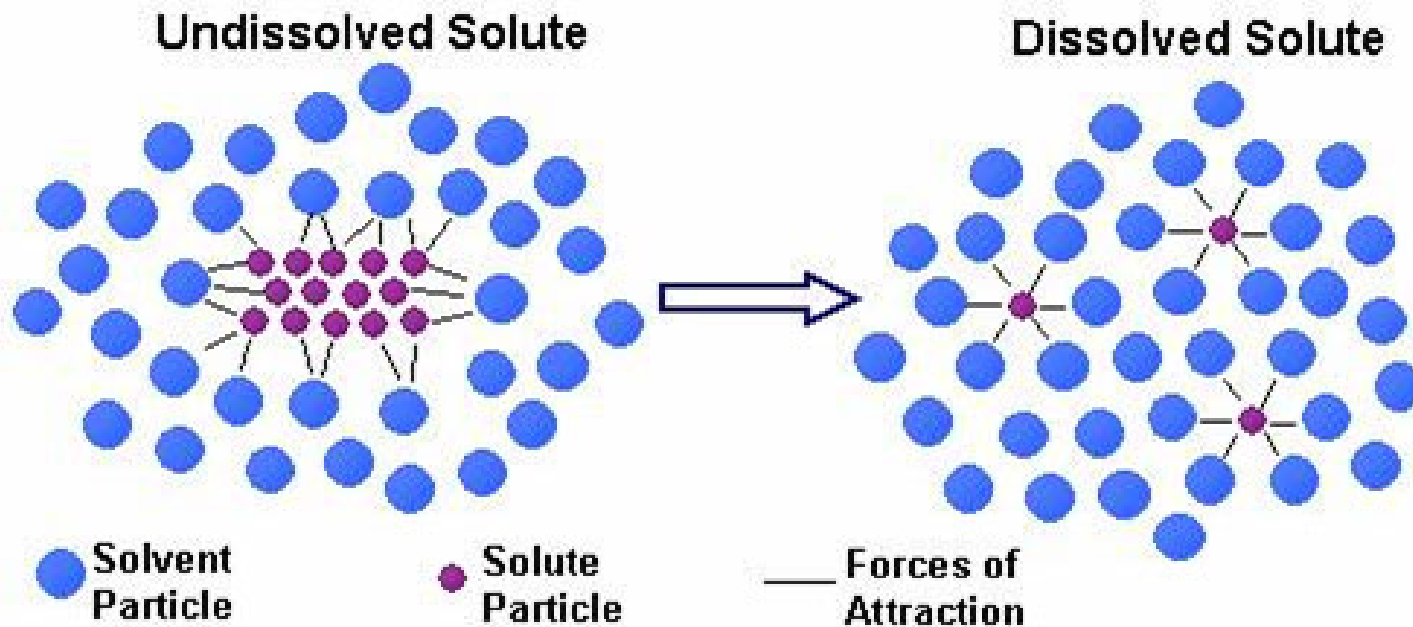
U. Domańska i M. Marciniak

$$x_i = \frac{1}{\gamma_i^L} \exp \left[\frac{\Delta h_i^{\text{talj}}}{R} \left(\frac{1}{T_i^{\text{talj}}} - \frac{1}{T} \right) - \frac{\Delta c_{pi}^{\text{talj}}}{R} \left(\ln \frac{T}{T_i^{\text{talj}}} + \frac{T_i^{\text{talj}}}{T} - 1 \right) \right]$$

$$T = \max(T_i)$$

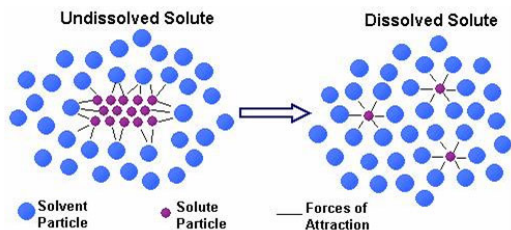
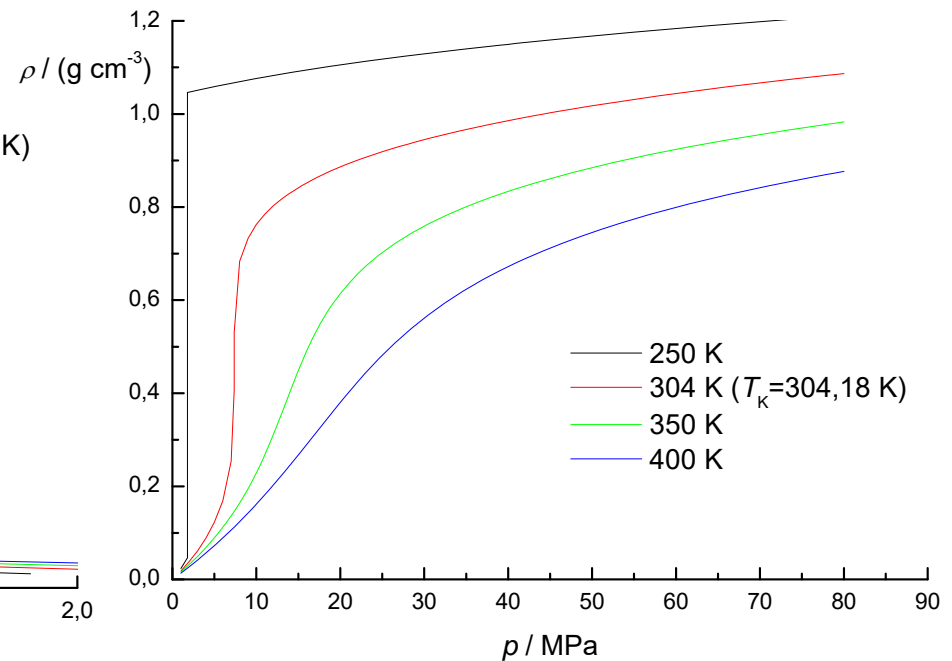
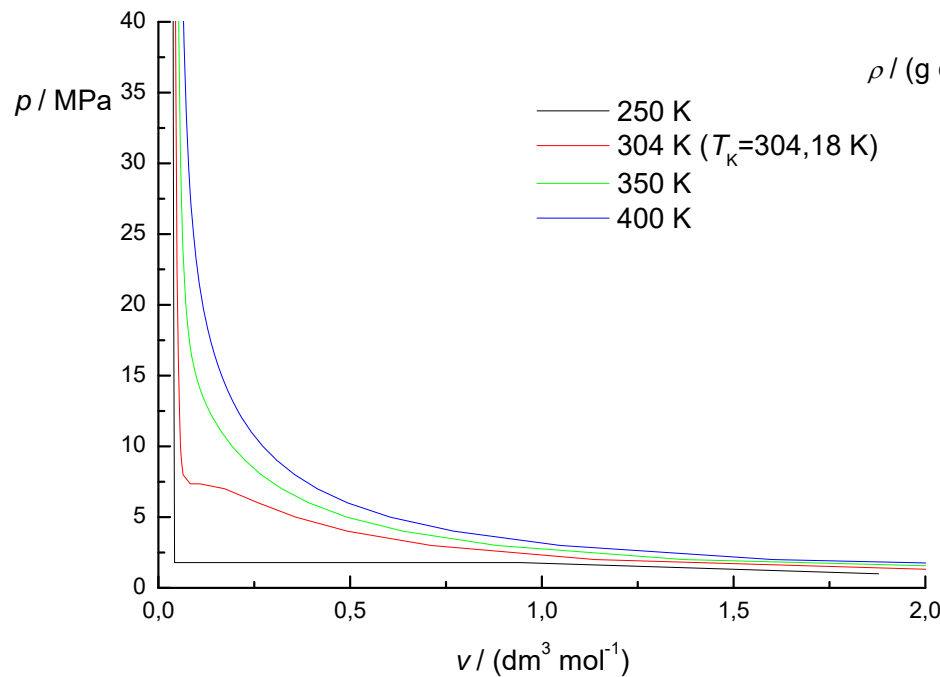


Ravnoteža plin-krutina



Ravnoteža plin-krutina

Nadkritični fluidi kao otapala



Velika gustoća – velika moć otapanja

Nema granice faza – bolja kinetika nego u kapljevinama

Niska temperatura – podobno za termolabilne ekstrakte

Laka *downstream* separacija

Nedostatak – slaba topljivost

Nedostatak – visoka cijena

Ravnoteža plin-krutina

Topljivost u nadkritičnom fluidu

$$\hat{f}_2^{\text{SF}} = \hat{f}_2^{\text{S}}$$

Jednadžba ravnoteže

Otopina

$$\hat{f}_2^{\text{SF}} = y_2^{\text{SF}} \hat{\phi}_2^{\text{SF}} p$$

$$\hat{f}_2^{\text{S}} = f_2^{\text{S}}$$

Čista krutina

Topljivost krutine u plinu

$$y_2^{\text{SF}} = \frac{1}{\hat{\phi}_2^{\text{SF}}} \cdot \frac{f_2^{\text{S}}}{p}$$

$$f_2^{\text{S}} = \phi_2^{\bullet} p_2^{\bullet} \exp\left[\frac{v_2^{\text{S}}(p - p_2^{\bullet})}{RT}\right]$$

Fugacitivnost čiste krutine

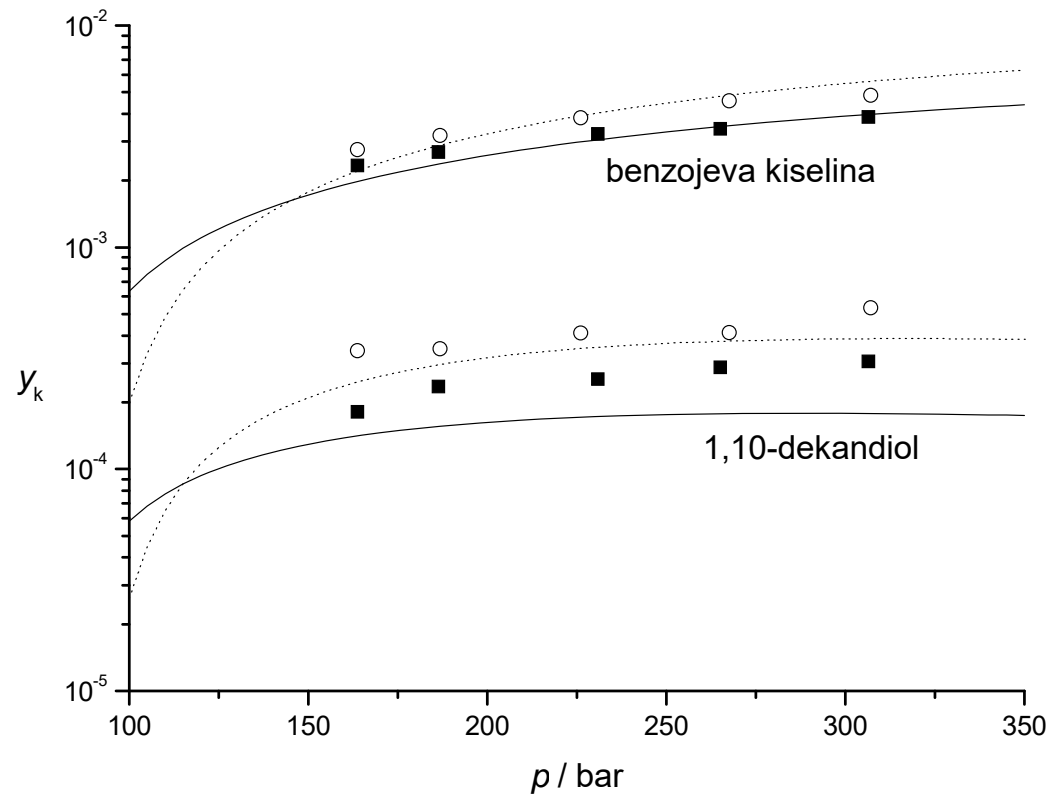
Topljivost krutine u plinu

$$y_2^{\text{SF}} = \frac{p_2^{\bullet}}{\hat{\phi}_2^{\text{SF},\infty} p} \exp\left[\frac{pv_2^{\text{S}}}{RT}\right]$$

$$y_2^{\text{SF}} = \frac{\phi_2^{\bullet} p_2^{\bullet}}{\hat{\phi}_2^{\text{SF}} p} \exp\left[\frac{v_2^{\text{S}}(p - p_2^{\bullet})}{RT}\right]$$

Pojednostavljenje

Ravnoteža plin-krutina



CO₂(1) – benzojeva kiselina(2) – 1,10-dekandiol(3)

M. Mukhopadhyay i G. V. Raghuram Rao – pravila miješanja

E. H. Chimovitz i K. J. Pennisi – podaci