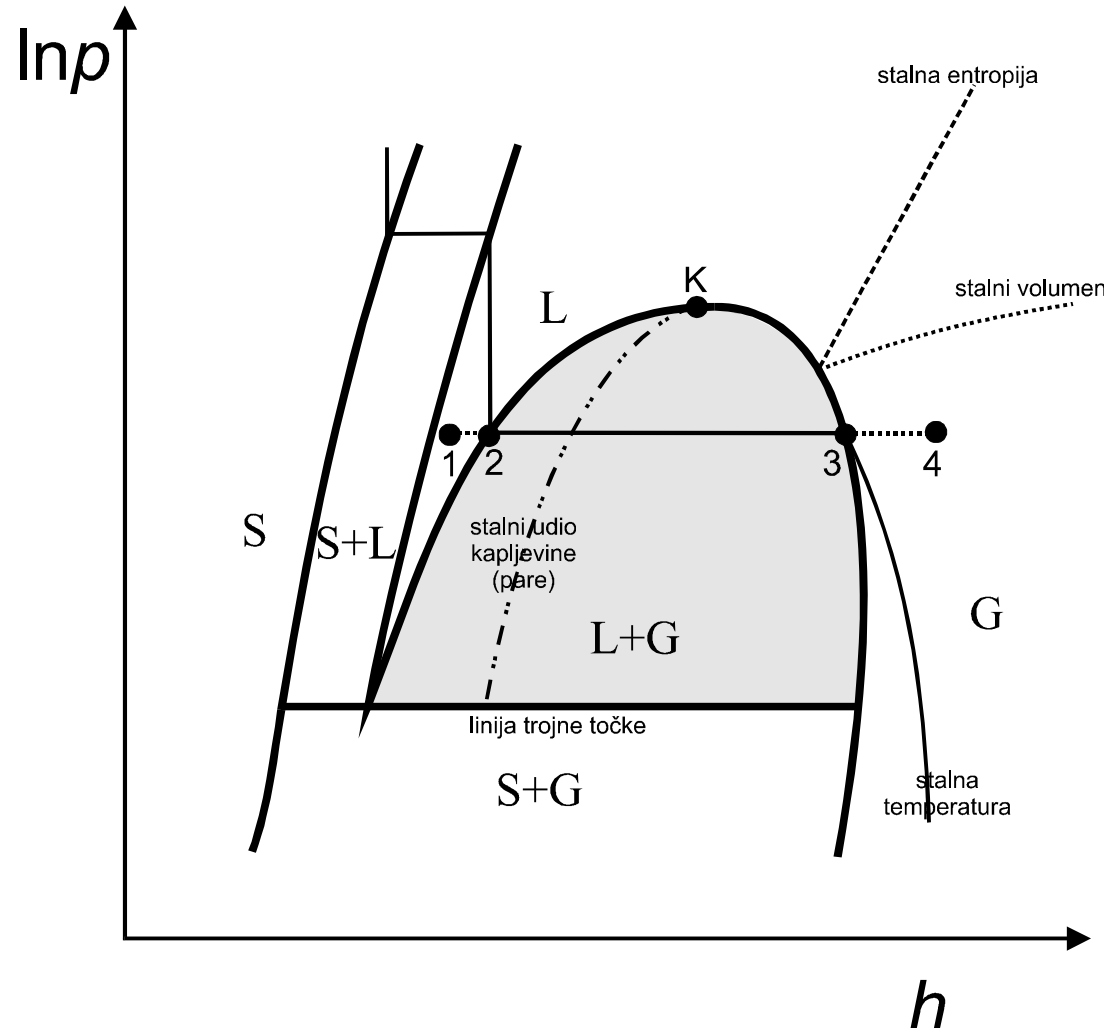
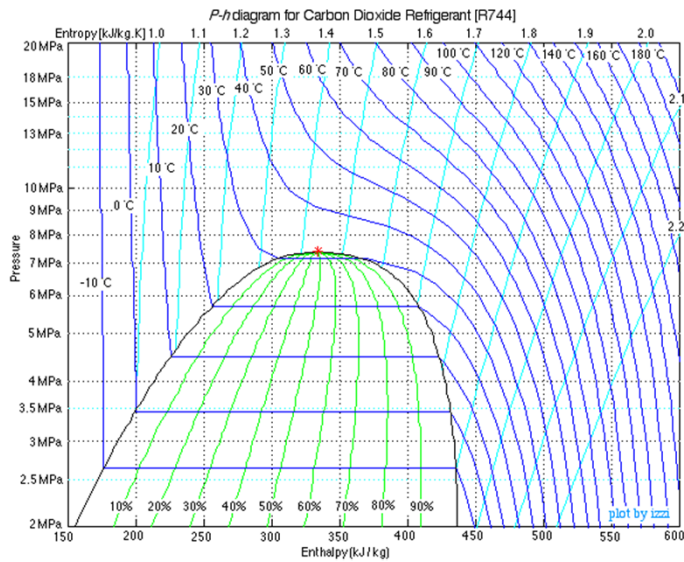


# Termodinamička svojstva realnih fluida

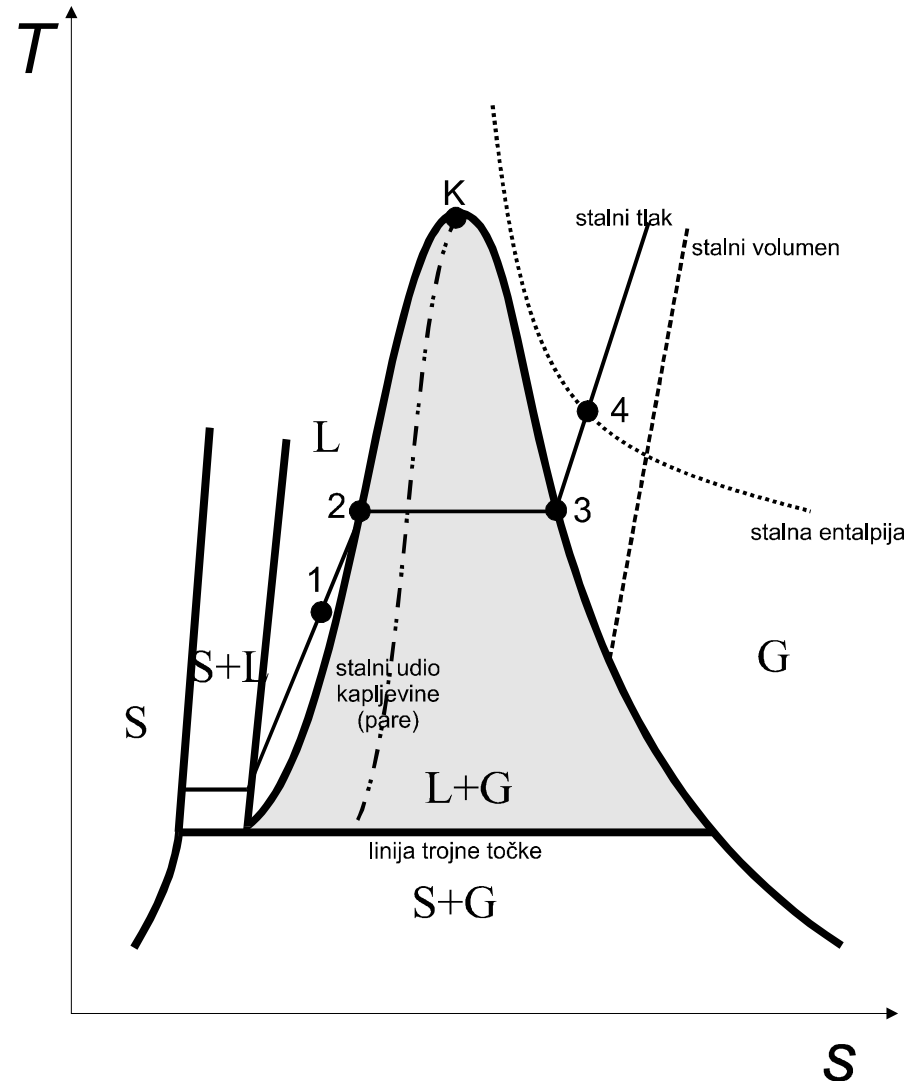
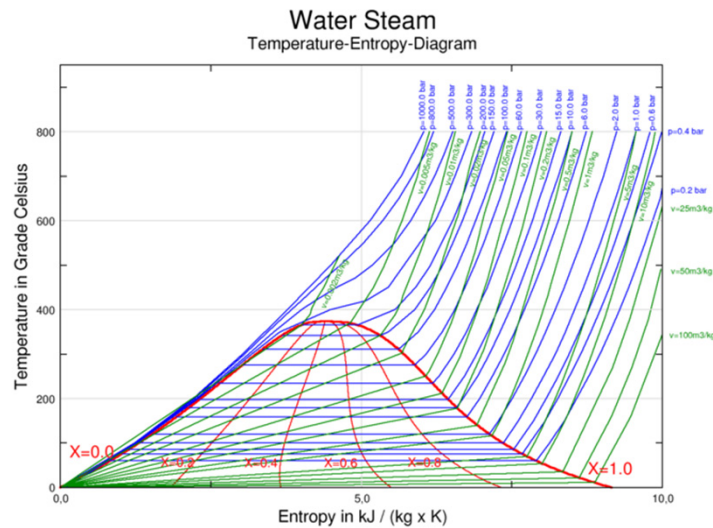
# Toplinske tablice i dijagrami

*ph*-dijagram  
Rashladni  
uređaji



# Toplinske tablice i dijagrami

Ts-dijagram  
Pretvorba  
Energije



# Toplinske tablice i dijagrami

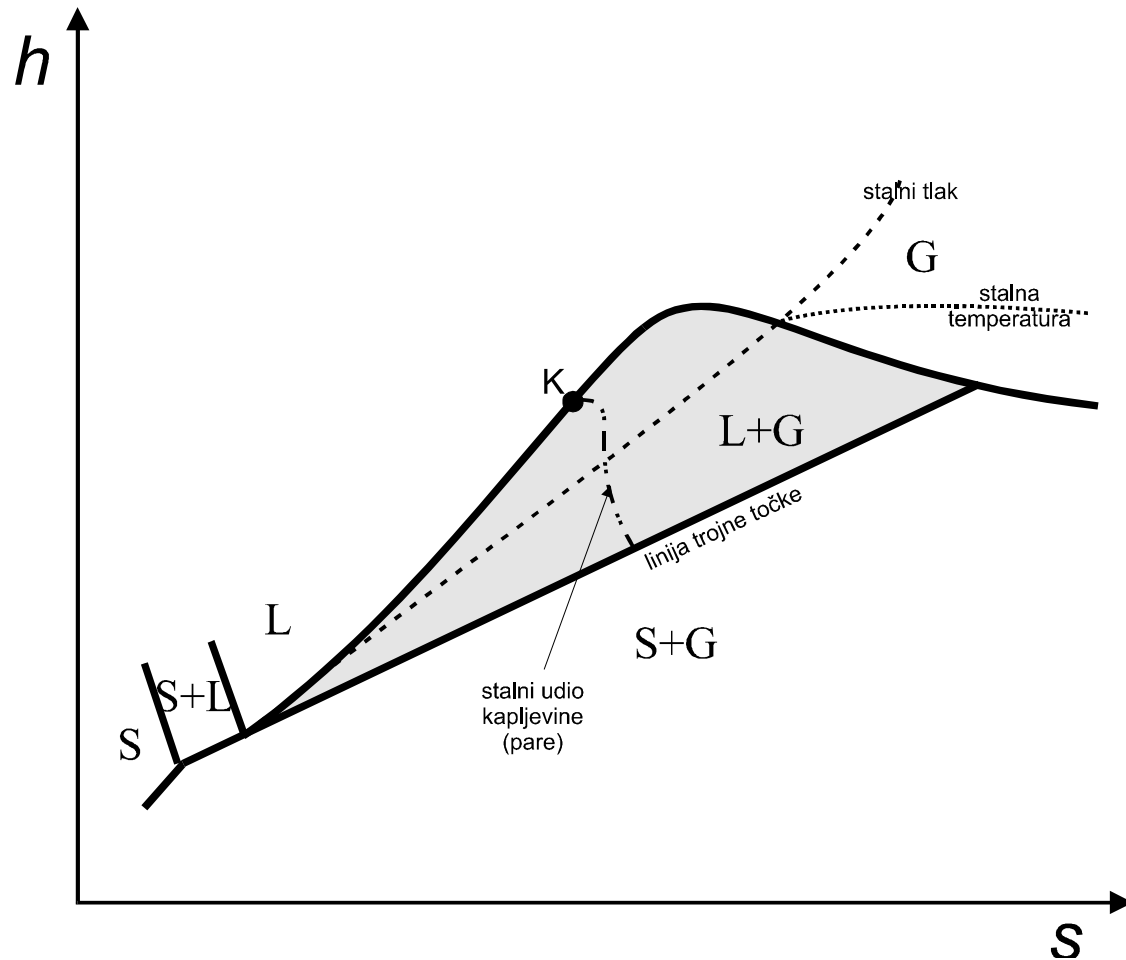
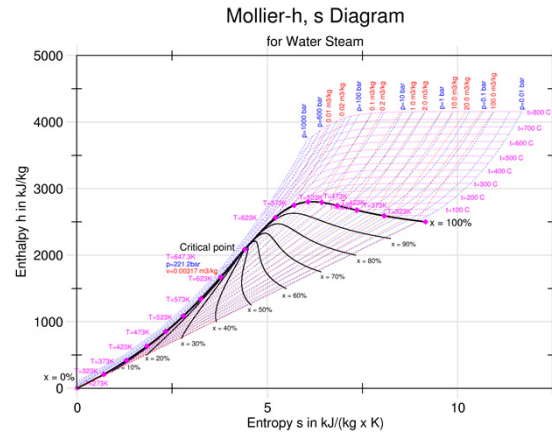
hs-dijagram

Mlaznice

Difuzori

Turbine

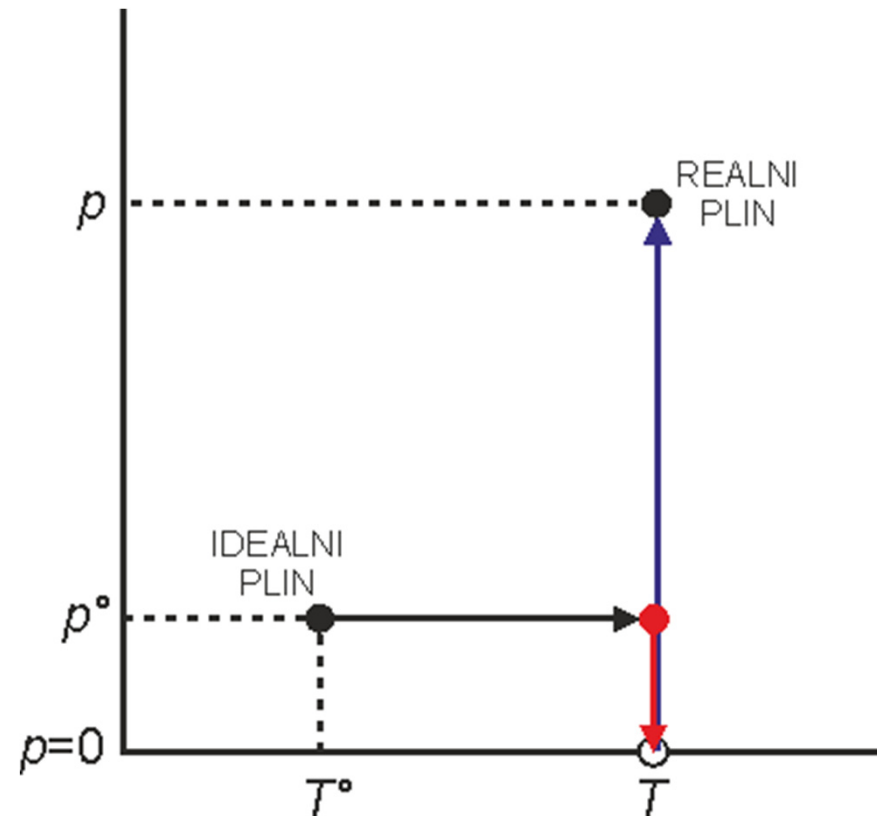
Kompresori



# Konstrukcija toplinskih dijagrama

Shema  
izračunavanja

- **izobarno zagrijavanje** idealnog plina od  $T^\circ$  do  $T$  pri referentnom tlaku  $p^\circ$
- **izotermna ekspanzija** idealnog plina pri  $T$  od referentnoga tlaka  $p^\circ$  do  $p=0$
- realni plin jednak idealnom pri  $p=0$
- **izotermna kompresija** realnog plina od  $p=0$  do  $p$  pri  $T$



# Konstrukcija toplinskih dijagrama

Entalpijska promjena

Temperatura  $\left(\frac{\partial h}{\partial T}\right)_p = c_p^{\text{id}} \quad \Delta_1 h = \int_{T^\circ}^T c_p^{\text{id}} dT$

$$c_p^{\text{id}} = a + bT + cT^2 + dT^3 + \dots$$

Tlak  $\left(\frac{\partial h}{\partial p}\right)_T = v - T\left(\frac{\partial v}{\partial T}\right)_p$

$$h - h^\circ = \int_{p^\circ}^0 \left[ v - T\left(\frac{\partial v}{\partial T}\right)_p \right] dp + \int_0^p \left[ v - T\left(\frac{\partial v}{\partial T}\right)_p \right] dp$$

$$h - h^\circ = \int_{p^\circ}^0 \left[ \frac{RT}{p} - T \frac{R}{p} \right] dp + \int_0^p \left[ v - T\left(\frac{\partial v}{\partial T}\right)_p \right] dp$$

Funkcija  
odstupanja  
*Deviation ili  
Departure Function*

$$h - h^\circ = 0 + \int_0^p \left[ v - T\left(\frac{\partial v}{\partial T}\right)_p \right] dp$$

# Konstrukcija toplinskih dijagrama

Entalpijska promjena

Za jednađbe stanja eksplicitne po

Funkcija  
odstupanja  
*Deviation ili  
Departure Function*

$$h - h^\circ = \int_0^p \left[ v - T \left( \frac{\partial v}{\partial T} \right)_p \right] dp \quad \text{volumenu}$$

$$h - h^\circ = RT(z - 1) + \int_\infty^v \left[ T \left( \frac{\partial p}{\partial T} \right)_v - p \right] dv \quad \text{tlaku}$$

Konačna entalpija:

$$h = h_{\text{ref}} + \int_{T^\circ}^T c_p^{\text{id}} dT + RT(z - 1) + \int_\infty^v \left[ T \left( \frac{\partial p}{\partial T} \right)_v - p \right] dv$$

# Konstrukcija toplinskih dijagrama

Entropijska promjena

Temperatura

$$\left(\frac{\partial s}{\partial T}\right)_p = \frac{c_p^{\text{id}}}{T} \quad \Delta_1 s = \int_{T^\circ}^T \frac{c_p^{\text{id}}}{T} dT$$

$$c_p^{\text{id}} = a + bT + cT^2 + dT^3 + \dots$$

Tlak

$$\left(\frac{\partial s}{\partial p}\right)_T = -\left(\frac{\partial v}{\partial T}\right)_p \quad s - s^\circ = \int_{p^\circ}^0 \left[-\left(\frac{\partial v}{\partial T}\right)_p\right] dp + \int_0^p \left[-\left(\frac{\partial v}{\partial T}\right)_p\right] dp$$

$$s - s^\circ = \int_{p^\circ}^0 \left[-\frac{R}{p}\right] dp - \int_0^p \left[\frac{R}{p}\right] dp + \int_0^p \left[\frac{R}{p}\right] dp + \int_0^p \left[-\left(\frac{\partial v}{\partial T}\right)_p\right] dp$$

Funkcija

odstupanja

*Deviation ili*

*Departure Function*

$$s - s^\circ - R \ln \frac{p}{p^\circ} + \int_0^p \left[\frac{R}{p} - \left(\frac{\partial v}{\partial T}\right)_p\right] dp$$



# Konstrukcija toplinskih dijagrama

Entropijska promjena

Za jednažbe stanja eksplicitne po

Funkcija  
odstupanja  
*Deviation ili*  
*Departure Function*

$$s - s^\circ = -R \ln \frac{p}{p^\circ} + \int_0^p \left[ \frac{R}{p} - \left( \frac{\partial v}{\partial T} \right)_p \right] dp \quad \text{volumenu}$$

$$s - s^\circ = R \ln \frac{v}{v^\circ} + \int_\infty^v \left[ \left( \frac{\partial p}{\partial T} \right)_v - \frac{R}{v} \right] dv \quad \text{tlaku}$$

Konačna entropija

$$s = s_{\text{ref}} + \int_{T^\circ}^T \frac{c_p^{\text{id}}}{T} dT + R \ln \frac{v}{v^\circ} + \int_\infty^v \left[ \left( \frac{\partial p}{\partial T} \right)_v - \frac{R}{v} \right] dv$$

# Funkcije odstupanja

$$h - h^\circ = RT(z - 1) + \int_{\infty}^v \left[ T \left( \frac{\partial p}{\partial T} \right)_v - p \right] dv$$

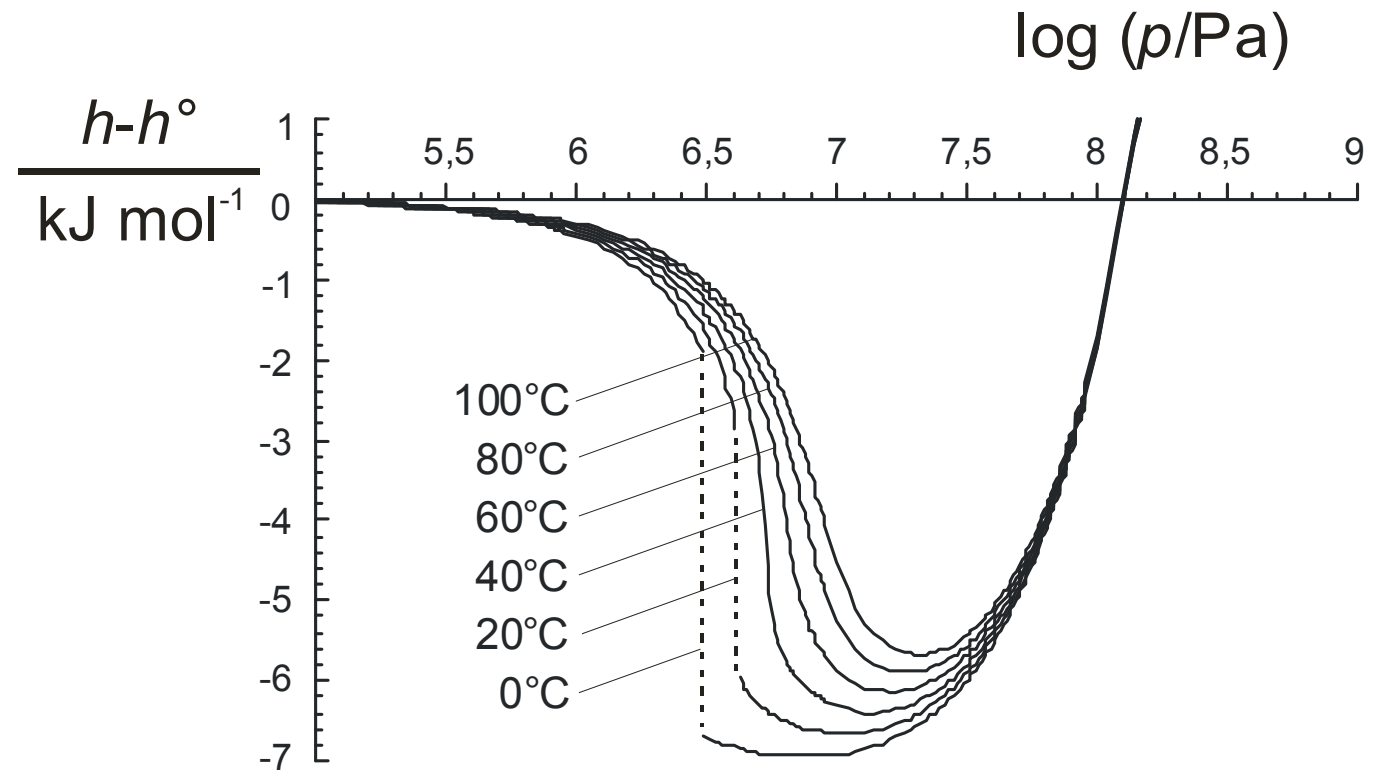
$$h - h^\circ = RT(z - 1) - \frac{a}{v}$$

$$p = \frac{RT}{v - b} - \frac{a}{v^2}$$

$$\left( \frac{\partial p}{\partial T} \right)_v = \frac{R}{v - b}$$

vdW

etan



# Funkcije odstupanja

$$s - s^\circ - R \ln \frac{p^\circ}{p} = R \ln z + \int_{\infty}^v \left[ \left( \frac{\partial p}{\partial T} \right)_v - \frac{R}{v} \right] dv$$

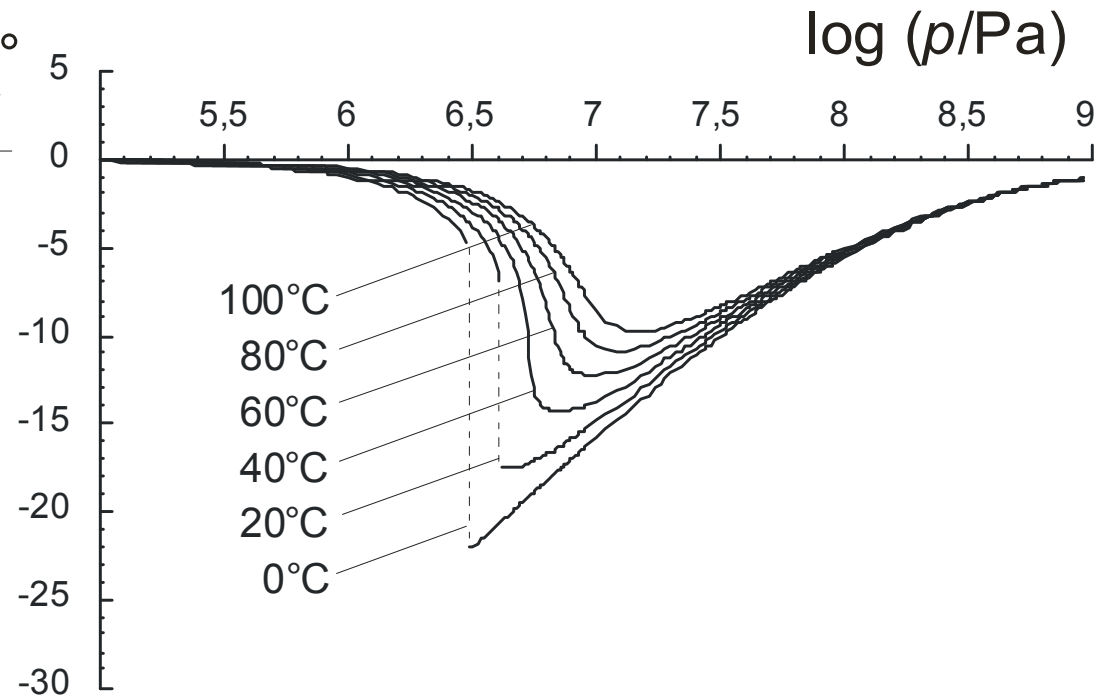
$$s - s^\circ - R \ln \frac{p^\circ}{p} = R \ln z + R \ln \frac{v-b}{v}$$

$$\left( \frac{\partial p}{\partial T} \right)_v = \frac{R}{v-b}$$

$$\frac{s - s^\circ - R \ln \frac{p^\circ}{p}}{\text{J mol}^{-1} \text{K}^{-1}}$$

vdW

etan



# Funkcije odstupanja

Peng-Robinsonova jednažba

$$s - s^\circ = -R \ln \frac{v^\circ}{v - b} + \frac{a}{b 2\sqrt{2}} \ln \frac{v + b(1 + \sqrt{2})}{v + b(1 - \sqrt{2})} \left( \frac{\partial \alpha}{\partial T} \right)_v$$

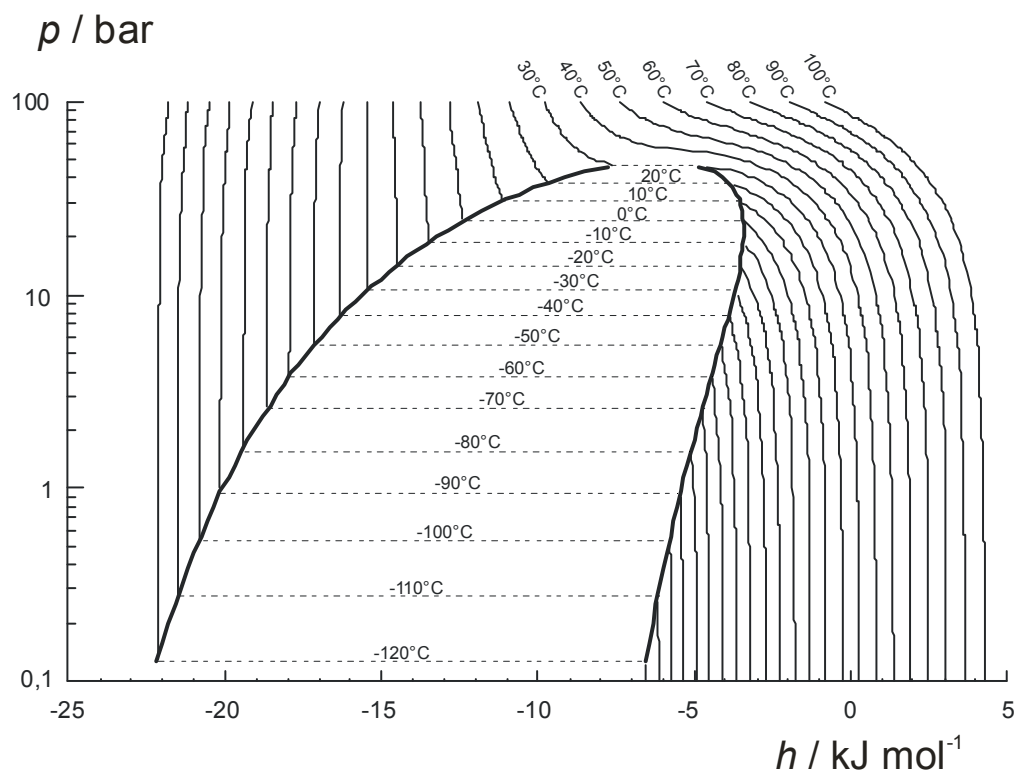
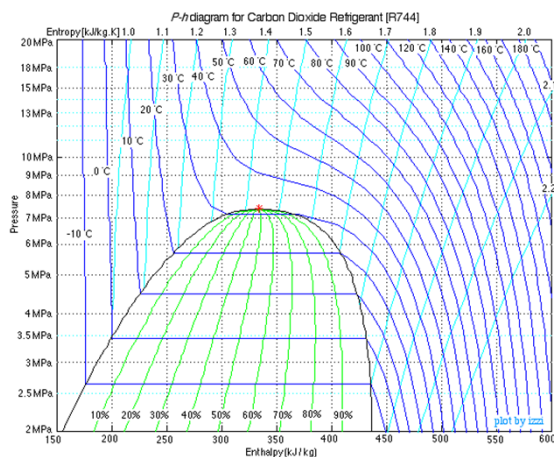
$$h - h^\circ = \frac{a}{b 2\sqrt{2}} \ln \frac{v + b(1 + \sqrt{2})}{v + b(1 - \sqrt{2})} \left[ T \left( \frac{\partial \alpha}{\partial T} \right)_v - \alpha \right] + RT(z - 1)$$

# Konstrukcija toplinskih dijagrama

Etan

$$T_K = 305,32 \text{ K}, p_K = 48,72 \text{ bar}, \omega = 0,099$$

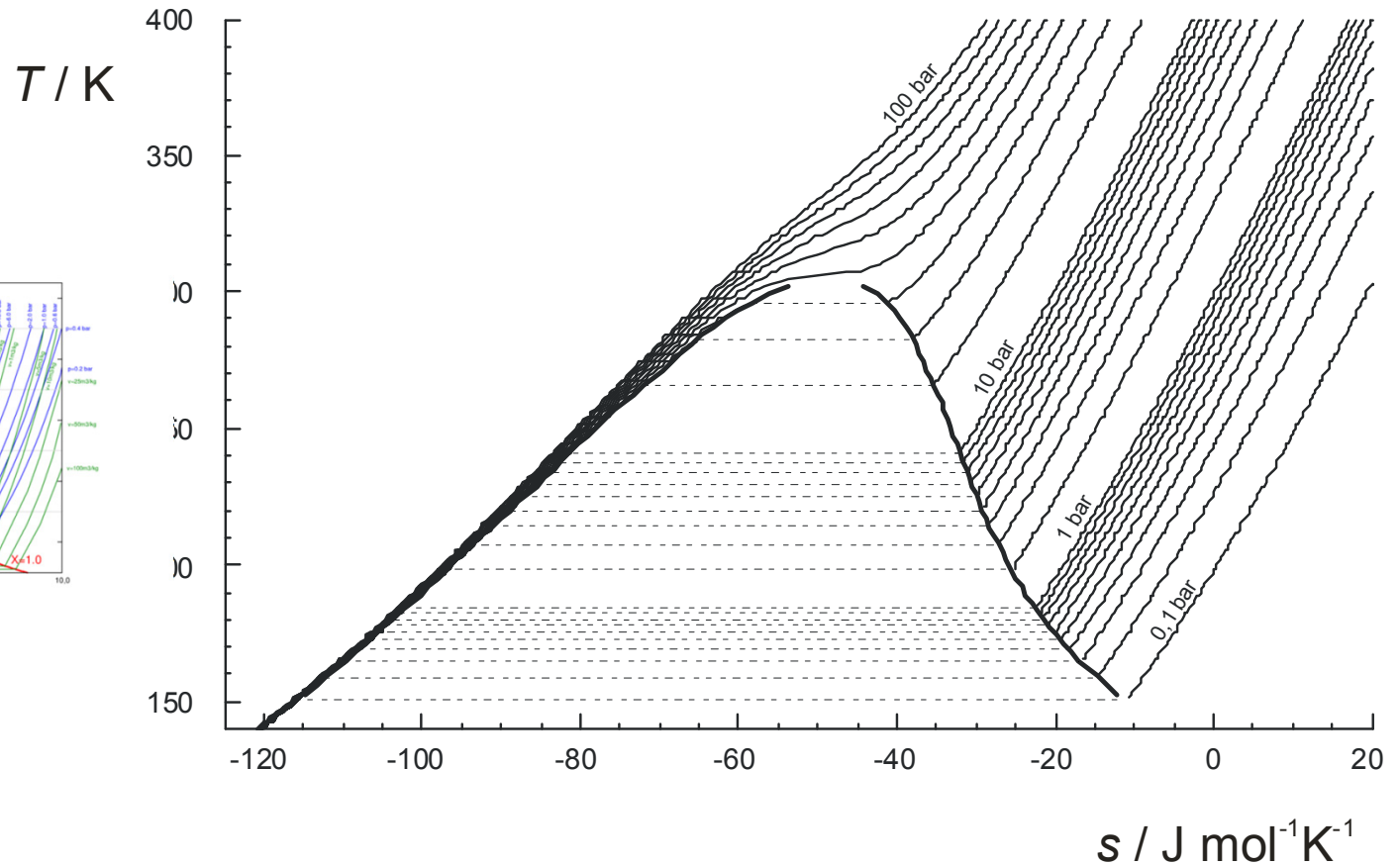
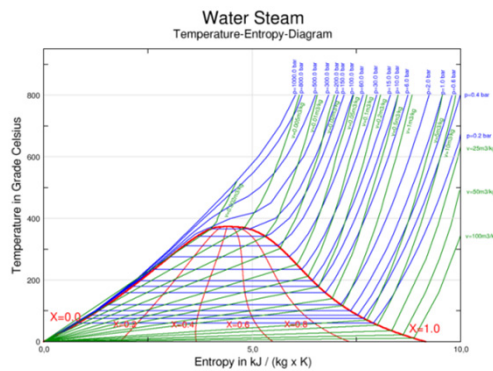
$$\frac{c_p^{\text{id}}}{R} = 4,178 - 4,427 \cdot 10^{-3} \frac{T}{\text{K}} + 5,660 \cdot 10^{-5} \left(\frac{T}{\text{K}}\right)^2 - 6,651 \cdot 10^{-8} \left(\frac{T}{\text{K}}\right)^3 + 2,487 \cdot 10^{-11} \left(\frac{T}{\text{K}}\right)^4$$



# Konstrukcija toplinskih dijagrama

Etan  $T_K=305,32$  K,  $p_K=48,72$  bar,  $\omega=0,099$

$$\frac{c_p^{\text{id}}}{R} = 4,178 - 4,427 \cdot 10^{-3} \frac{T}{\text{K}} + 5,660 \cdot 10^{-5} \left(\frac{T}{\text{K}}\right)^2 - 6,651 \cdot 10^{-8} \left(\frac{T}{\text{K}}\right)^3 + 2,487 \cdot 10^{-11} \left(\frac{T}{\text{K}}\right)^4$$



# Načelo korespondentnih stanja

$$h - h^\circ = \int_0^p \left[ v - T \left( \frac{\partial v}{\partial T} \right)_p \right] dp$$

$$s - s^\circ = \int_0^p \left[ \frac{R}{p} - \left( \frac{\partial v}{\partial T} \right)_p \right] dp$$

$$\frac{h - h^\circ}{T_K} = -RT_r^2 \int_0^{p_r} \left[ \frac{1}{p_r} \left( \frac{\partial z}{\partial T_r} \right)_{p_r} \right] dp_r$$

$$s - s^\circ = R \int_0^{p_r} \left[ \frac{1 - z}{p_r} - \frac{T_r}{p_r} \left( \frac{\partial z}{\partial T_r} \right)_{p_r} \right] dp_r$$

# Načelo korespondentnih stanja

$$h - h^\circ = \int_0^p \left[ v - T \left( \frac{\partial v}{\partial T} \right)_p \right] dp$$

$$s - s^\circ = \int_0^p \left[ \frac{R}{p} - \left( \frac{\partial v}{\partial T} \right)_p \right] dp$$

$$\frac{h - h^\circ}{RT_K} = -T_r^2 \int_0^{p_r} \left[ \frac{1}{p_r} \left( \frac{\partial z}{\partial T_r} \right)_{p_r} \right] dp_r$$

$$s - s^\circ = R \int_0^{p_r} \left[ \frac{1 - z}{p_r} - \frac{T_r}{p_r} \left( \frac{\partial z}{\partial T_r} \right)_{p_r} \right] dp_r$$

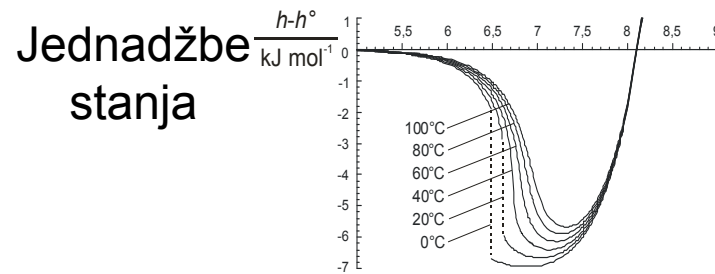
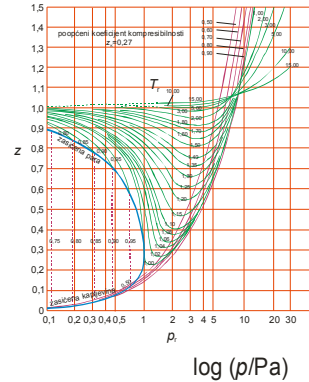
$$\frac{h - h^\circ}{T_K} = f(p_r, T_r)$$

$$s - s^\circ = f(p_r, T_r)$$



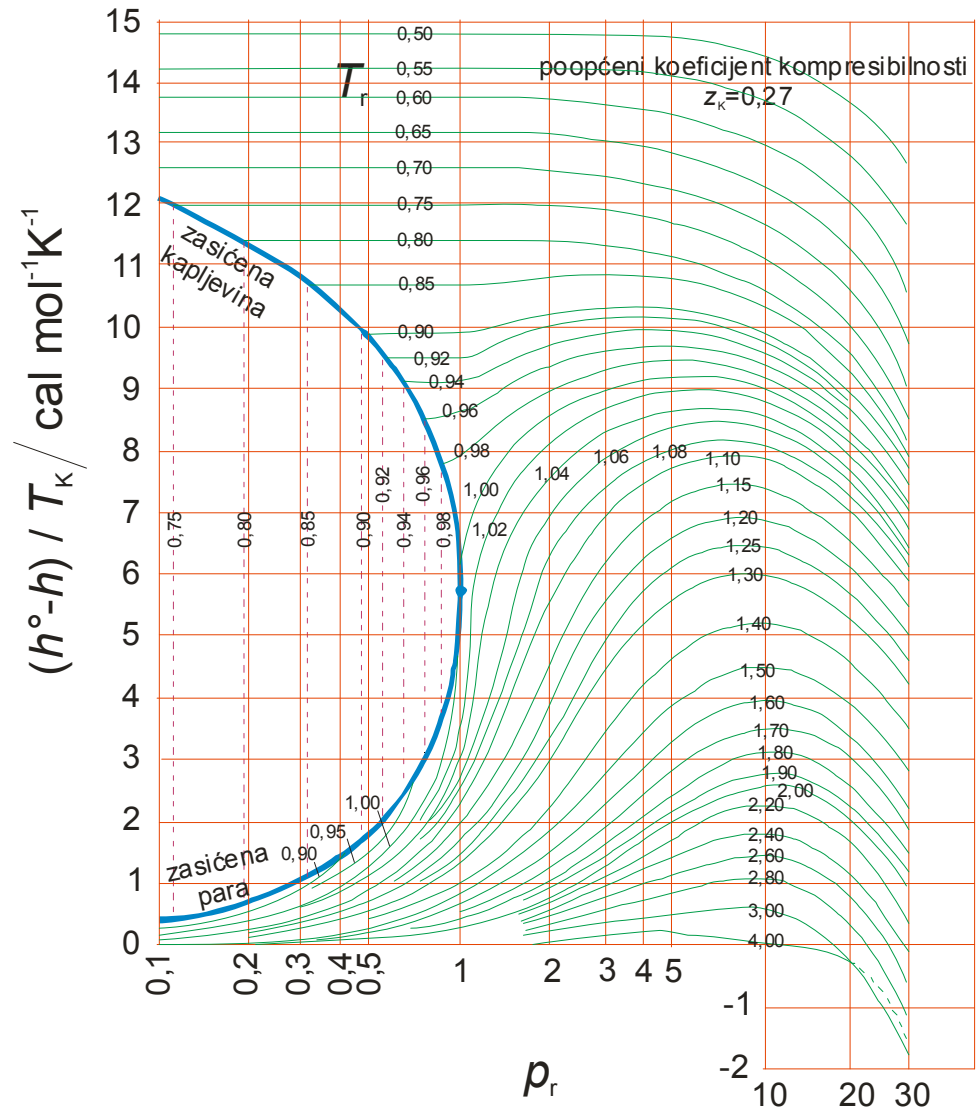
# Načelo termodinamičke sličnosti

$$z = f(p_r, T_r, z_K)$$



$$\frac{h - h^\circ}{T_K} = f(p_r, T_r, z_K)$$

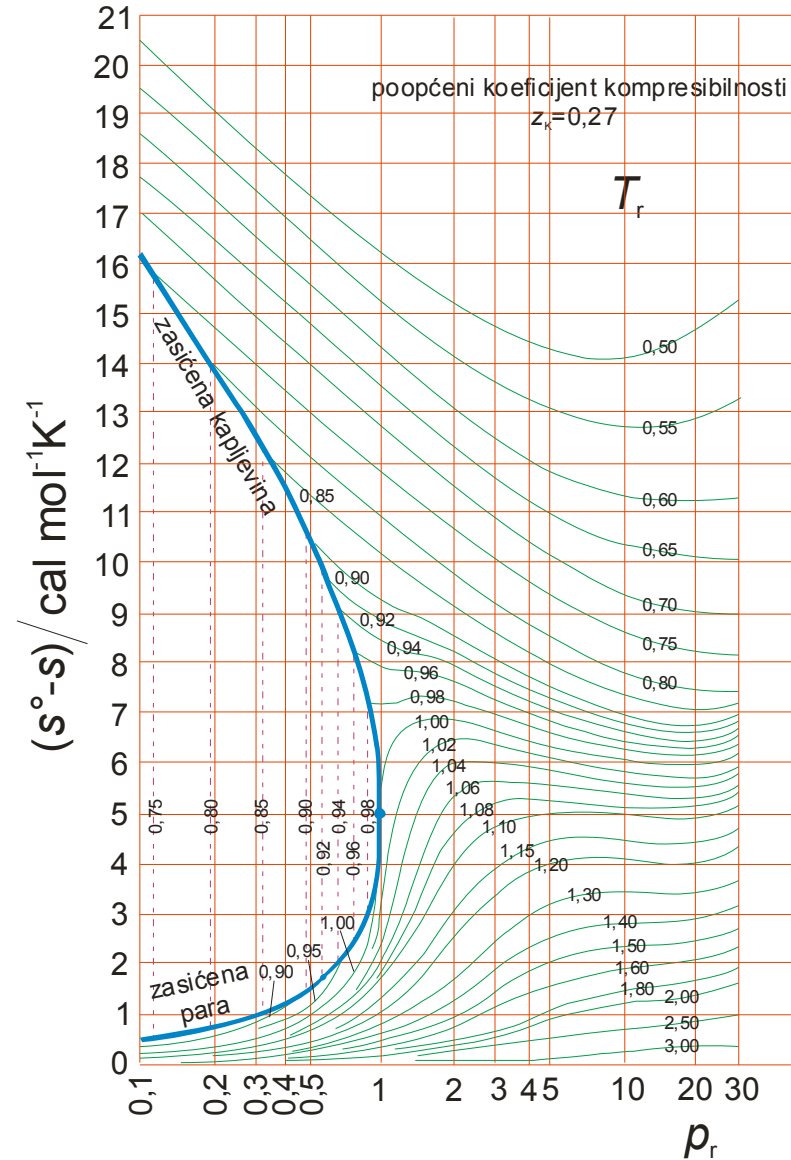
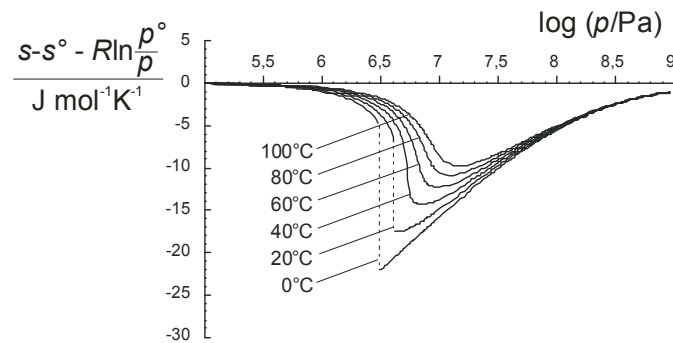
$$s - s^\circ = f(p_r, T_r, z_K)$$



# Načelo termodinamičke sličnosti

$$s - s^\circ = f(p_r, T_r, z_K)$$

Jednadžbe stanja

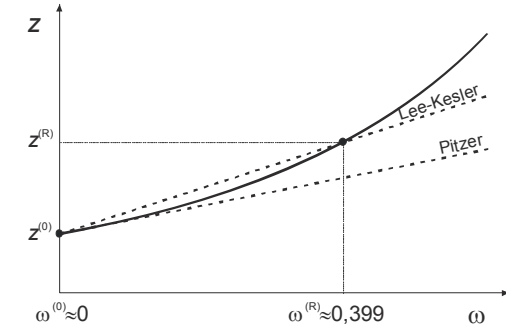


# Načelo termodinamičke sličnosti

Lee-Keslerova korelacija (1975)

$$z = f(p_r, T_r, \omega)$$

$$z = z^{(0)}(T_r, p_r) + \omega z^{(1)}(T_r, p_r)$$



$$\left( \frac{h^\circ - h}{RT_K} \right) = \left( \frac{h^\circ - h}{RT_K} \right)^{(0)} + \omega \left( \frac{h^\circ - h}{RT_K} \right)^{(1)}$$

$$\left( \frac{s^\circ - s}{R} \right) = \left( \frac{s^\circ - s}{R} \right)^{(0)} + \omega \left( \frac{s^\circ - s}{R} \right)^{(1)} - \ln \frac{p^\circ}{p}$$

# Fugacitivnost

Zatvoreni sustavi,  $p, T = \text{konst}$

$$g = h - Ts = u + pv - Ts$$

$$dg = vdp - sdT$$

$$(dg)_T = vdp$$

$$(dg)_T^{\text{id}} = \frac{RT}{p} dp = RT d \ln p$$

G. N. Lewis (1901)

$$(dg)_T = vdp = RT d \ln f$$

Fugacitivnost je tlak koji bi realni plin imao kada bi se vladao kao idealan ???

„Convenience functions” Sandler  
„Zgodne, prikladne funkcije”



Logaritamsko računalo (šiber)

# Koeficijent fugacitivnosti

Razlika realnog i idealnog plina

$$(dg)_T - (dg)_T^{\text{id}} = RTd \ln f - RTd \ln p$$

$$d(g - g^{\text{id}}) = RTd \ln \frac{f}{p}$$

$$\int_{g - g^{\text{id}}(p=0)}^{g - g^{\text{id}}(p)} d(g - g^{\text{id}}) = RT \int_{\ln(f/p)(p=0)}^{\ln(f/p)(p)} d \ln \frac{f}{p}$$

$$g - g^{\text{id}} = RT \ln \frac{f}{p}$$

$$\lim_{p \rightarrow 0} \frac{f}{p} = 1$$

G. N. Lewis (1901)

Koeficijent fugacitivnosti

$$\varphi = \frac{f}{p}$$

„Convenience functions” Sandler  
„Zgodne, prikladne funkcije”

# Izračunavanje fugacitivnosti iz jednadžbi stanja

Definicijski izraz – volumetrijska svojstva

$$(dg)_T = vdp = RTd \ln f$$

$$RT \left( \frac{\partial \ln f}{\partial p} \right)_T = v$$

Jednadžbe stanja eksplicitne po

volumenu

$$f = p \exp \left[ \frac{1}{RT} \int_0^p \left( v - \frac{RT}{p} \right) dp \right]$$

$$\ln \phi = \int_0^p (z - 1) d \ln p$$

tlaku

$$f = p \exp \left[ (z - 1) - \ln z + \frac{1}{RT} \int_{\infty}^v \left( \frac{RT}{v} - p \right) dv \right] \quad \ln \phi = \frac{1}{RT} \int_{\infty}^v \left( \frac{RT}{v} - p \right) dp + (z - 1) - \ln z$$

# Izračunavanje koeficijenta fugacitivnosti iz jednadžbi stanja

$$\ln \varphi = \ln \frac{v}{v-b} + \frac{a}{bRT^{3/2}} \ln \frac{v}{v+b} + (z-1) - \ln z \quad \text{RK}$$

$$\ln \varphi = \ln \frac{v}{v-b} + \frac{a\alpha(T)}{bRT} \ln \frac{v}{v+b} + (z-1) - \ln z \quad \text{SRK}$$

$$\ln \varphi = \ln \frac{v}{v-b} - \frac{a\alpha}{bRT 2\sqrt{2}} \ln \frac{v+b(1+\sqrt{2})}{v+b(1-\sqrt{2})} + (z-1) - \ln z \quad \text{PR}$$

# Izračunavanje Gibbsove energije iz fugacitivnosti

Iznos Gibbsove energije ovisi o izboru referentnog stanja

Preko funkcija odstupanja (u odnosu na idealni plin pri 1 bar i 25 °C)

$$g = h - Ts \quad h = h_{\text{ref}} + \int_{T^\circ}^T c_p^{\text{id}} dT + RT(z-1) + \int_{\infty}^v \left[ T \left( \frac{\partial p}{\partial T} \right)_v - p \right] dv$$
$$s = s_{\text{ref}} + \int_{T^\circ}^T \frac{c_p^{\text{id}}}{T} dT + R \ln \frac{v}{v^\circ} + \int_{\infty}^v \left[ \left( \frac{\partial p}{\partial T} \right)_v - \frac{R}{v} \right] dv$$

Preko fugacitivnosti (u odnosu na realni plin pri 1 bar i temperaturi sustava)

Standardno stanje plina – stanje realnog plina pri 1 bar i temperaturi sustava

$$f^\circ = p^\circ = 1 \text{ bar (nekad jedna atmosfera)}$$



# Izračunavanje Gibbsove energije iz fugacitivnosti

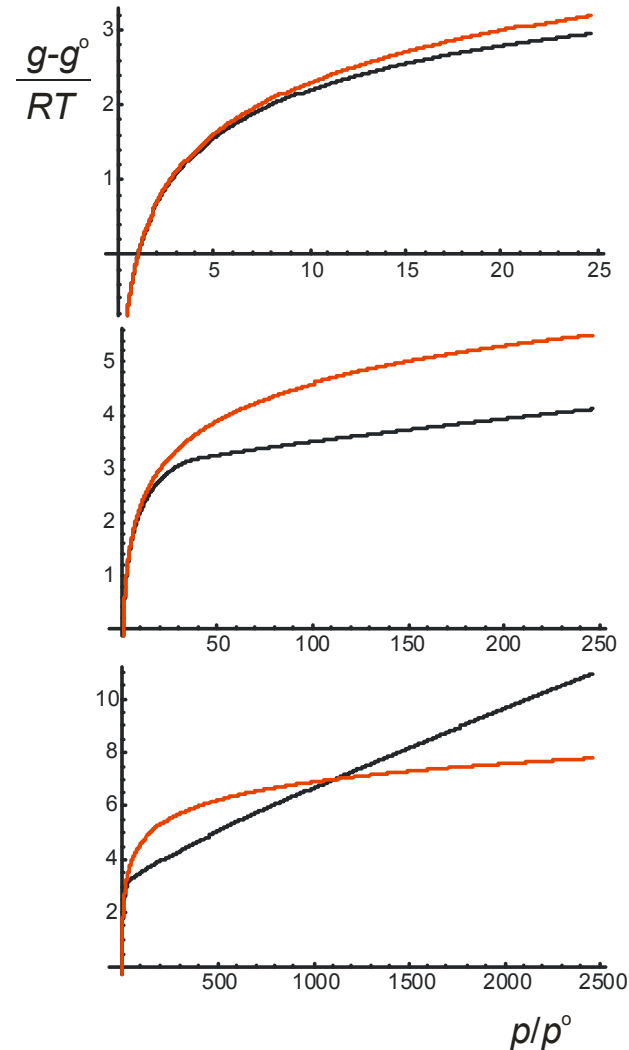
Idealni plin

$$g = g^\circ + RT \ln \frac{p}{p^\circ}$$
$$\frac{(g - g^\circ)^{\text{id}}}{RT} = \ln \frac{p}{p^\circ}$$

SRK, dušik

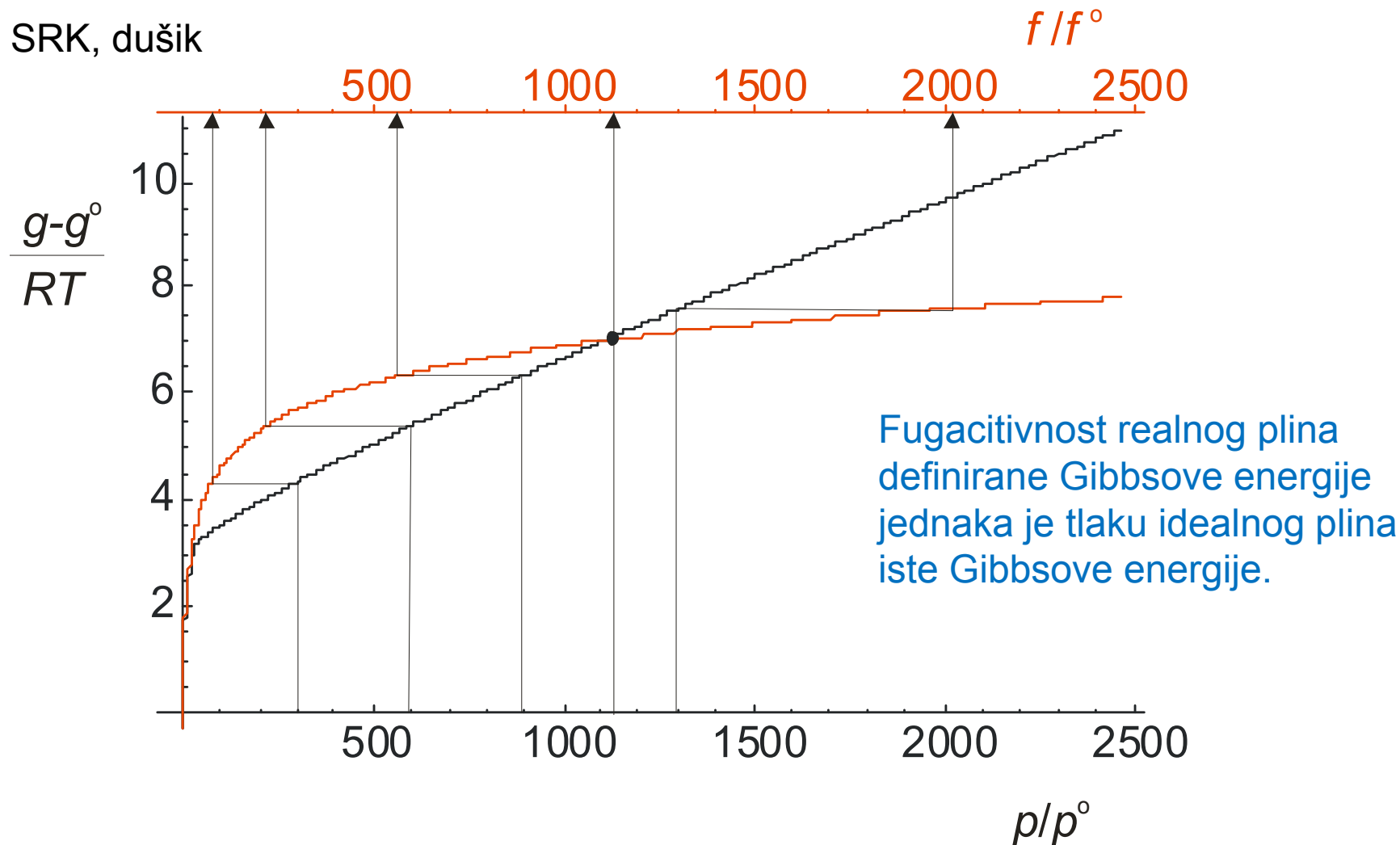
$$\frac{g - g^\circ}{RT} = \ln \frac{f}{f^\circ}$$

$$\frac{g - g^\circ}{RT} = \ln \frac{v^\circ}{v - b} + \frac{a\alpha}{bRT} \ln \frac{v}{v + b} + (z - 1)$$

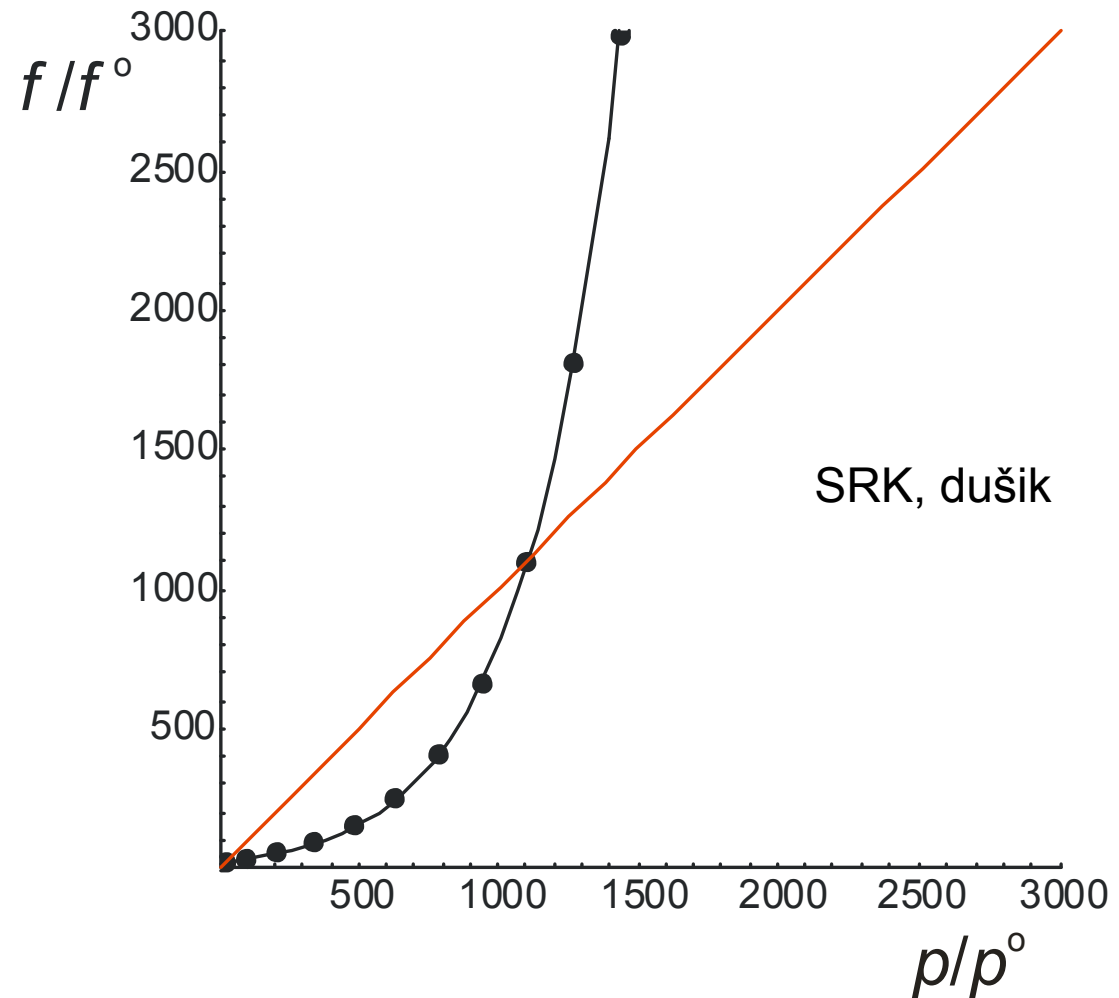
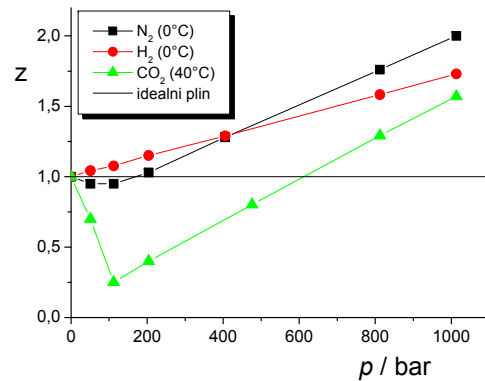
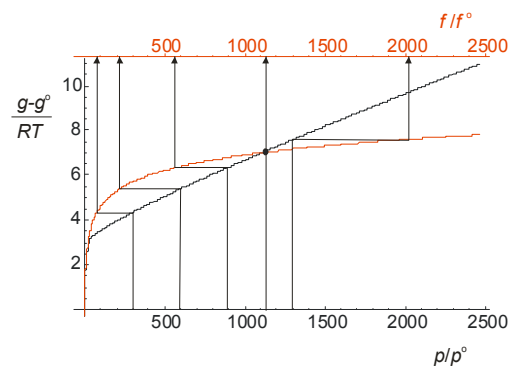


# Izračunavanje Gibbsove energije iz fugacitivnosti

SRK, dušik



# Izračunavanje Gibbsove energije iz fugacitivnosti



# Načelo usporedivih stanja

$$\ln \varphi = \int_0^p \frac{(z-1)}{p} dp$$

$$\ln \varphi = \int_0^p \frac{(z-1)}{p_r} dp_r$$

Dijagrami i tablice

$$\ln \varphi = f(p_r, T_r)$$

# Načelo termodin. sličnosti

Lee-Kesler  $\ln \varphi = f(p_r, T_r, \omega)$

$$\ln \varphi = \ln \varphi^{(0)} + \frac{\omega}{\omega^{(R)}} \left( \ln \varphi^{(R)} - \ln \varphi^{(0)} \right)$$

$$z = \frac{v}{v_{id}}$$

$$\ln \varphi = \ln \varphi^{(0)} + \omega \ln \varphi^{(1)}$$

$$z = z^{(0)}(T_r, p_r) + \omega z^{(1)}(T_r, p_r)$$

$$\varphi = \frac{f}{p}$$

$$\left( \frac{h^\circ - h}{RT_K} \right) = \left( \frac{h^\circ - h}{RT_K} \right)^{(0)} + \omega \left( \frac{h^\circ - h}{RT_K} \right)^{(1)}$$

$$\left( \frac{s^\circ - s}{R} \right) = \left( \frac{s^\circ - s}{R} \right)^{(0)} + \omega \left( \frac{s^\circ - s}{R} \right)^{(1)} - \ln \frac{p^\circ}{p}$$